The Association

of

Engineering and Shipbuilding Draughtsmen.

The Elements of Reinforced Concrete Design.

By H. V. CRABTREE (Member).

Published by The Technical Section, Association of Engineering and Shipbuilding Draughtsmen, 96 St. George's Square, London, S.W. 1.

SESSION 1925-26.

Milne, Tannahill, & Methven, 12-14 Mill Street, Perth.

ADVICE TO INTENDING AUTHORS OF A.E.S.D. PRINTED LECTURES.

Lectures for submission to the National Technical Sub-Committee for consideration for publication in this series should normally approximate to 10,000 words and about 20 illustrations. (A Lecture of this size makes up into a pamphlet of about 40 pages).

Drawings for illustrations should be done either on a good plain white paper or tracing cloth, deep black indian ink being used. For ordinary purposes they should be made about one-and-a-half times the intended finished size, and it should be arranged that wherever possible this shall not be greater than a single full page of the pamphlet, as folded pages are objectionable, although, upon occasions, unavoidable. Where drawings are made larger involving a greater reduction the lines should be made slightly heavier and the printing rather larger than normal, as the greater reduction tends to make the lines appear faint and the printing excessively small in the reproduction. In the case of charts or curves set out on squared paper, either all the squares should be inked in or the chart or curve should be retraced and the requisite squares inked in.

Manuscripts, in the first instance, should be submitted to the General Secretary, 96 St. George's Square, Westminster, London, S.W.1., and marked " Technical Section."

For Lectures of normal size a grant of £10 is made to the Author. Special consideration is given in the case of larger ones involving more than the usual amount of preparation.

The A.E.S.D. accepts no responsibility for the formula or opinions expressed in its Technical Publications.

The Association

of

Engineering and Shipbuilding Draughtsmen.

The Elements of Reinforced Concrete Design.

By H. V. CRABTREE (Member).

Published by The Technical Section, Association of Engineering and Shipbuilding Draughtsmen, 96 St. George's Square, London, S.W. 1.

SESSION 1925-26.

Milne, Tannahill, & Methven, 12-14 Mill Street, Perth.

THE ELEMENTS OF REINFORCED CONCRETE DESIGN.

By H. V. CRABTREE (Member).

Reinforced or Ferro Concrete is a combination of Concrete reinforced with steel bars in such a manner that the steel

(1) Takes all the tensile forces in the combination.

(2) Helps the concrete in the resistance of shear and compression where necessary.

The idea of reinforcing concrete with metal is not a new one. Such early engineers as the Romans leave us examples of this method of construction, but it is doubtful whether they had much knowledge of the mechanics involved in the design, although they most certainly had some idea of the fundamental principle, that is to reinforce the concrete where it is weakest, viz., in the tensile regions. Their materials, however, were very crude, Portland Cement and Steel being unknown.

With the advent of the cheap manufacture of Portland Cement and Steel, along with the awakening of engineers to the possibilities of Reinforced Concrete, and the untiring energy of the pioneers of this construction during the latter part of the last and the beginning of this century, **Reinforced Concrete has become one of the leading, if not the leading method of construction throughout the world.**

It is used for all types of construction, such as Buildings, Bridges, Piers, Sewers, Dams, Reservoirs, Bunkers, Roads, Piles, etc., the one notable exception to success being its application to shipbuilding.

For the purpose of this article, the author has adhered strictly to the limits and methods allowed by the London County Council in their By-Laws relating to Reinforced Concrete. These By-laws have recently been revised, which should make the following more interesting. The L.C.C. is the leading authority with legislative powers in this country, although their regulations undoubtedly err on the side of safety.

MATERIALS AND WORKING STRESSES.

Concrete.—The type of concrete used for reinforced concrete work is a mixture of Portland cement, sharp sand, and broken stone, brick or other hard ballast, and water.

The **Cement** should be in accordance with the British Standard specification, ninety pounds of cement being deemed the equivalent of one cubic foot for measuring purposes.

The **Sand** should be pit or river sand, sharp and clean, and should be graded in size up to particles which will pass a three-sixteenth inch mesh.

Shore or sea sand is not to be desired, as the particles are rounded and are too even, thus not being able to grip and lock with each other.

Washed sand is much to be preferred, if obtainable, especially as the latter is usually screened and thus allows of a suitable selection being made for grading purposes.

Ballast should be pit ballast, or broken stone or rock, granite being the ideal ballast. The same remarks that apply to the washing and grading of sand apply equally well for the ballast.

The following materials though often used are not suitable for the ballast for reinforced concrete work and are definitely disqualified by the L.C.C. Clinkers, ashes, coke breeze, pan breeze, furnace slag, limestones of any description or any other similar materials.

The ballast should be graded in size from that which will pass through a $\frac{3}{4}$-in. square mesh down to a $\frac{3}{16}$-in. mesh, but it should not be larger than will pass between the reinforcing bars or between the bars and the centering. Thus in the case of say a floor slab with only $\frac{1}{2}$-in. of concrete covering under the bars, the ballast would not have to exceed that which will pass a $\frac{1}{2}$-in. mesh.

It is advisable to wet the ballast before mixing the concrete, especially if it is of a porous nature. This prevents the moisture from the mixture of the sand, cement and water which is usually termed the mortar, from being sucked into the ballast.

Too much cannot be said for the thorough grading of the sand and ballast in such a manner that there is a minimum of voids, and the volume of mortar should be slightly in excess of that required to fill the voids.

The proportions of the concrete are measured by volume. The volume of sand should in no case exceed twice the volume of cement, and the volume of the ballast should not exceed twice that of the sand, in fact, it is usual to fix the proportion of the ballast twice that of the sand.

It is also usual to speak of a mixture of concrete in the proportions of the cement, sand and ballast, thus a mixture of one part of cement, two parts sand, and four parts ballast would be termed a 1 : 2 : 4 mixture, or a 1 : 6 mixture.

The strength of concrete at first rapidly increases, then more slowly afterwards. The strength two months after setting is ap-

proximately 25% greater than that at one month, and at four months about 50% greater than at one month, after which the increase is very small, and it is practically negligible after twelve months.

The usual working stresses are assumed at a period of four months after setting, and the following is a list of safe compressive stresses for varying mixtures, as allowed by the L.C.C. Regulations, provision having been made for a factor of safety of four.

MIXTURE.	SAFE COMPRESSIVE STRESS.
1 : 2 : 4 or 1 : 6	600 lbs. per square inch.
1 : $1\frac{2}{3}$: $3\frac{1}{3}$ or 1 : 5	650 ,, ,, ,, ,,
1 : $1\frac{1}{3}$: $2\frac{2}{3}$ or 1 : 4	700 ,, ,, ,, ,,
1 : 1 : 2 or 1 : 3	750 ,, ,, ,, ,,

The value for intermediate mixtures may be obtained from the following formula :—

$$c = 900 - 50\,v$$

where c = safe compressive resistance in lbs. per square inch,
and v = volume of sand and ballast per unit volume of cement.

It will be observed that the tabulated values conform with this equation.

The safe shearing stresses allowed are one-tenth of the safe compressive stresses.

No allowance whatever is made for the tensile strength of concrete, as it is so very small and unreliable.

The value of the grip or adhesion of concrete to steel is taken at 100 lb. per square inch of the surface of the steel bar, when same is effectively anchored by bending or otherwise securing at the end, and 60 lb. when not effectively anchored.

All the foregoing values err slightly on the safe side, so that it should hardly be necessary to make any tests, unless demanded by the authorities, to ascertain that the concrete being used is capable of withstanding the stresses.

It is common practice in districts outside the jurisdiction of the L.C.C. or other authorities to increase the foregoing values for the strength of concrete by 15% to 20%, but when this is done, tests should be made on sample cubes or cylinders about six inches each way, to verify the concrete will withstand the increased stresses.

Steel.—The reinforcement should be made of mild steel of quality in accordance with British Standard Specification No. 15 for structural steel for bridges and general building construction.

The ends of all tensile and shear reinforcement should be hooked or otherwise effectively anchored in the concrete, or bent over a main reinforcing bar where convenient.

The reinforcement should never be painted, as this prevents the adhesion of the concrete.

A slight coating of rust is not detrimental, as this assists the adhesion, but a thick coating of rust, or any grease or dirt should be removed. A wash of cement grout is very advantageous, as it prevents undue corrosion and greatly assists the adhesion.

The safe tensile stress in the steel reinforcement as allowed by the L.C.C. Regulations is 16,000 lb. per square inch. The stress in compressive reinforcement depends on the Modular Ratio of Elasticity of the steel and concrete described in the next paragraph.

Designers outside the limits of Authority areas frequently use a safe working limit of 18,000 lb. tensile stress per square inch, and sometimes even up to 25,000 lb. per square inch.

MODULUS OF ELASTICITY AND MODULAR RATIO.

The Modulus of Elasticity of any material is the " unit stress/unit strain " or assuming that the material would not break and that the strain was constantly proportional to the stress, it is the stress that would stretch the material to twice its original length. This is an assumption which could never be an established fact, as the material would break long before its length was doubled, with perhaps the exception of such elastic materials as rubber, etc. A compressive strain is assumed as a negative elongation, and as a matter of fact the tests for the elasticity of concrete are carried out in compression, those for steel being made on tensile test pieces.

For steel, the strain is fairly proportional to the stress until the elastic limit is reached and a " stress/strain diagram " would be a straight line graph. The modulus of elasticity of steel is approximately 30,000,000 lb. per square inch.

The strain in concrete is not proportional to the stress but accelerates with an increase in stress, giving a " stress/strain " graph of approximately a parabolic line. For the purposes of fixing a modulus of elasticity this is determined from tests made with stresses about the safe working limits of the concrete, and the assumption is then made that the modulus of elasticity is a constant figure within the working limits. This errs slightly on the safe side in some cases, as will be seen when we treat the flexure of beams. The modulus of elasticity of concrete is about 2,000,000 for a 1 : 6 mixture of concrete and increases up to 2,500,000 lb. per square inch for a 1 : 3 mixture or it is about $3{,}333\frac{1}{3} \times c$, where c = the safe compressive stress in the concrete.

The ratio of the two values is termed the Modular Ratio (m) and will be as follows :—

$$m = \frac{30{,}000{,}000}{3{,}333\frac{1}{3}\,c} = \frac{9{,}000}{c}$$

$\therefore$ m varies from 15 for a 1 : 6 mixture to 12 for a 1 : 3 mixture.

Fig. 1.

It follows that steel working in conjunction with concrete must have a stress in proportion to the modular ratio of the materials. That is, if a bar of steel is placed in a piece of concrete, as shewn in Fig. 1, and compressed longitudinally the stress in the steel will be (m) times the stress in the concrete, the allowable stresses having nothing whatever to do with the actual stresses. Therefore the maximum stress that can be put in compressive reinforcement will be $c \times m =$ 9000 lb. per square inch, because as c increases on account of the mixture there is a proportionate decrease in the value of m. As 9000 per square inch is little more than half the working limit of steel, it follows that it is economical to reduce the compressive reinforcement to a minimum.

SHORT SIMPLE REINFORCED PILLARS OR COLUMNS.

Notation Used.

A = Effective area of the pillar in square inches.
A_v = Total area of the vertical reinforcement in square inches.
c = Permissible direct compressive stress per square inch on the concrete.
d = Effective diameter or width in inches.
m = Modular ratio.
P = Permissible load on short pillar.
r_v = Ratio of area of vertical reinforcement to effective area.

Short columns are those which would fail by direct crushing. When the length of a column exceeds about fifteen times its narrowest effective width (d) in the case of a rectangular column or twelve times its effective diameter (d) in the case of a round column, a crippling tendency occurs in addition to the direct compressive stress. More will be said of long columns later, and we will now deal with short columns.

Fig. 2 shews three typical sections of pillars. The effective area is shewn etched, and is measured (like the effective diameter) to the outer edge of the vertical reinforcement. The cover is assumed to be for protective purposes only and is not taken into account in the calculations. The L.C.C. Regulations call for the cover in a

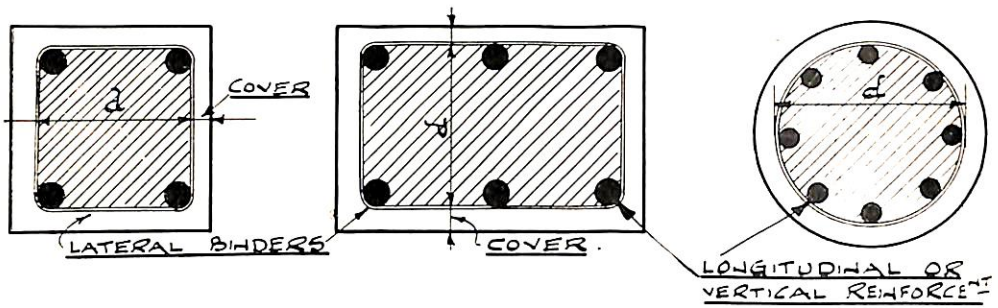


FIG 2.

pillar to be not less than $1\frac{1}{2}$ in. and not less than the diameter of the vertical reinforcement.

Rectangular pillars should have not less than four, and circular pillars not less than six vertical reinforcement bars. The net diameter of such bars should not be less than $\frac{1}{2}$-in. nor more than 2 in. and the total cross sectional area should not be less than 1 per cent. of the effective area of the pillar.

The lateral binding in simple reinforced columns is merely to keep the vertical reinforcement in position and is wrapped round or otherwise secured to same.

The binders should be not more than $0{\cdot}6d$ apart and at each end of the pillar for a length of $1\frac{1}{2}d$ they should not be more than $0{\cdot}3d$ apart.

Rectangular or rectilinear laterals as they are termed should not be less than 3/16-in. in diameter.

Circular or curvilinear laterals should not be less than 1/8-in. in diameter.

The volume of lateral reinforcement should not be less than 0·5% of that of the hooped core of concrete, viz., that within the lateral binding.

Following the reasoning that the stress in the vertical reinforcement is m times the stress in the concrete, we get the formulae :—

$$P = Ac + mA_v c - A_v c$$

$$\text{or} \quad P = c\,[A + (m - 1)\,A_v]$$

All stresses being in lbs. per square inch and all areas in square inches. Example :—To find the load (P) which may be carried by a square 1 : 6 concrete pillar with an effective width of 15 in. and reinforced with four vertical bars 1 in. in diameter.

Effective area of pillar A = 15 × 15 = 225 square inches.

Total area of reinforcement $A_v = 4 \times \cdot7854 =$ say 3·14 sq. ins.

c for 1 : 6 concrete = 600 $\qquad m = \dfrac{9000}{600} = 15$

$P = c\,[A + (m-1)\,A_v] = 600\,[225 + 14 \times 3{\cdot}14] = 161{,}376$ lbs.

Usually, however, the value of A_v is worked as a proportion of the value of A and thus $A_v = Ar_v$.

Substituting this value for A_v in the formulae we get

$$P = c\,[A + (m-1)\,Ar_v] = cA\,[1 + (m-1)\,r_v]$$

$$\text{or} \quad A = \frac{P}{c\,[1 + (m-1)\,r_v]}$$

this allows us to work the required area for any given load.

Example :—To find the necessary effective area of a short pillar using a $1 : 1\frac{1}{3} : 2\frac{2}{3}$ mixture of concrete and 2 per cent. of vertical reinforcement to take a load of 124,000 lb.

$$c = 700 \qquad r_v = \cdot02 \qquad m = \frac{9000}{700} = \text{say } 13$$

$$A = \frac{P}{c\,[1 + (m-1)\,r_v]} = \frac{124{,}000}{700\,[1 + \cdot24]} = 142 \text{ sq. ins.}$$

say 12 in. × 12 in.

The area of the reinforcement would be 2% of 142 square inches = 2·84.

Four bars 1-in. diam. would give a total area of 3·14 square inches, which would be very suitable.

The pillar would be as shewn in Fig. 3, the lateral binding being spaced at ·6d centres and ·3d at the ends as previously stated, to keep the vertical reinforcement in position.

SHORT PILLARS OR COLUMNS WITH HOOPED CORE.

If any material is stressed a strain or deflection is caused ; thus if we put a tensile stress in the material we stretch it and an increase in length occurs, or if we compress it there is a reduction in length. But corresponding with this increase or reduction in length there is also a slight lateral deflection. When there is an increase in length there is a decrease in width, and when there is a reduction in length there is an increase in width.

To more fully illustrate what is meant, let us assume that we compress the cylinder of concrete shewn in Fig. 4 in the direction of the arrows. Under compression it will become shorter and slightly barrel shaped as shewn exaggerated in Fig. 4a, and if it were compressed until it crushed, the actual bursting would be laterally. Now if we bind that piece of concrete by wrapping it with steel or

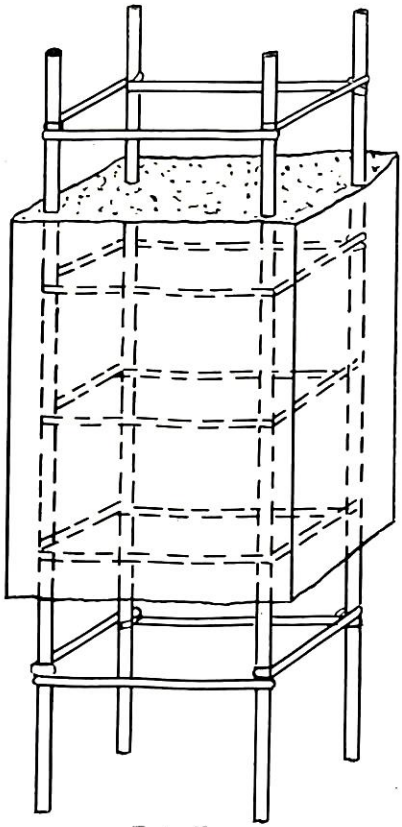


FIG 3

FIG. 4.

FIG. 4A.

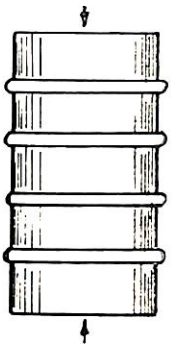
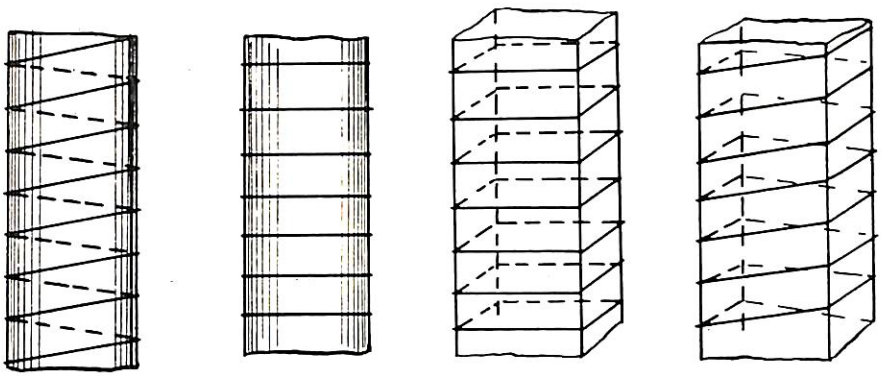


FIG. 4B.

placing steel hoops round it, as shewn in Fig. 4b, the lateral deflection will be restricted, which in turn will restrict the longitudinal deflection and thus increase the strength of the cylinder. (M. Considère invented the lateral form of reinforcement).

The increase in strength due to lateral reinforcement depends upon three factors, viz. : (1) The ratio of the volume of reinforcement to the volume of hooped core. This ratio is indicated by (V_1) in our calculations. (2) The form (f) of binding used, and (3) the spacing of the binders (s). There are three types of binding—(1) Helical (Curvilinear in plan), (2) Circular hoops, and (3) Rectilinear, as shewn in Fig. 5.



(1) HELICAL. (2) CIRCULAR HOOPS. (3) RECTILINEAR.

FIG 5.

It will be observed that there are two types of binding under the heading of rectilinear, but for the purposes of calculation they are the same.

The increased resistance (i) due to lateral binding is found from the following formula which is more or less empirical.

$$i = c\ (1 + f.s.\ V_1)$$

The form factor (f) is 1·0 for helical, 0·75 for circular hoops and 0·5 for either form of rectangular reinforcement.

The spacing factor may be found from the formula :—

$$s = 48 - 80\,\frac{P_b}{d}$$

where P_b = the pitch of the binders.

A list is given on the next page shewing varying values of f and s.

Form of Lateral Reinforcement.	Form Factor *f*.	Pitch of Laterals.	Spacing Factor *s*.	Value of *fs*.
HELICAL	1·0	0·2d (or less)	32	32
,,	1·0	0·3d	24	24
,,	1·0	0·4d	16	16
CIRCULAR HOOPS	0·75	0·2d (or less)	32	24
,, ,,	0·75	0·3d	24	18
,, ,,	0·75	0·4d	16	12
RECTILINEAR	0·5	0·2d (or less)	32	16
,,	0·5	0·3d	24	12
,,	0·5	0·4d	16	8
,,	0·5	0·5d	8	4
,,	0·5	0·6d	0	0

The spacing factor should not be taken at more than 32 if P_b is less than $0{\cdot}2d$ but intermediate values for the spacing factor may be obtained from the formula given.

The value of V_1 (neglecting any slight increase in length due to the diagonal inclination of the binders) will be :—

$$V_1 = \frac{4\ A_b}{d\ P_b} \text{ for either round or square columns.}$$

Having found the value of (i) we can substitute same for c in the formulae for simple reinforced columns which then becomes :—

$$P = i\ [A + (m-1)\ A_v] \text{ or } P = iA\ [1 + (m-1)\ r_v]$$

$$\text{and } A = \frac{P}{i\ [1 + (m-1)\ r_v]}$$

In this case we work the value of m from i instead of c.

$$\therefore\ m = \frac{9000}{i}$$

Example :—To design a short square section column to take a load of 248,000 lb. having 2% of vertical reinforcement and 1·5% of lateral binding spaced at $0{\cdot}3d$ pitch the mixture being 1 : 6.

$i = c\ (1 + fsV_1) = 600\ (1 + 12 \times 0{\cdot}015) =$ say 700 lbs. per sq. in.

$m = \frac{9000}{700} =$ say 13 $\quad\therefore$ Reqd. $A = \frac{248{,}000}{700\ (1 + 12 \times {\cdot}02)} =$ 285 square inches, say 17 inches square.

The area of the verticals would be 2% of 275 = 5·5 sq. in., say four rounds $1\frac{3}{8}$-in. diameter.

The area of the binders would be obtained from the following :—

$$V_1 = \frac{4\ A_b}{d\ P_b} \text{ or } A_b = \frac{V_1\ d P_b}{4} = \frac{0{\cdot}015 \times 17 \times 17 \times 0{\cdot}3}{4}$$

$$= 0{\cdot}325 \text{ sq. in.}$$

this would require an 11/16-in. diam. bar, which while being rather heavy suits the purpose for our calculations.

It should be stated that the value of i must in no case exceed $1\frac{1}{3}$ times the value of c to comply with the L.C.C. regulations, no matter how much lateral binding is used, but this value is often and quite reasonably exceeded. Some engineers often exceed the ultimate compressive resistance of plain concrete when laterally reinforced.

PILLARS SUBJECT TO BENDING STRESSES.

Very often pillars or columns are subject to bending stresses in addition to the direct compressive stress.

Bending stresses may be caused either by eccentric loading or by lateral thrusts such as those set up by wind pressures.

To design such a pillar it is necessary to combine the direct and bending stresses, the total of which must not exceed the permissible stress in the material.

The direct stress (f_d) will be found from the formula

$$f_d = \frac{P}{A\ [1 + (m-1)\ r_v]}$$ in accordance with the reasoning set forth for short columns.

The bending stress (f_b) is determined from the formula $f_b = \frac{B}{Z}$

where B = the bending moment, and Z = the section modulus of the pillar about the plane of bending.

Combining the two we get a fibre stress f =

$$\frac{P}{A\ [1 + (m-1)\ r_v]} + \frac{B}{Z}$$

at the maximum compression edge of the pillar,

and, $f = \frac{P}{A\ [1 + (m-1)\ r_v]} - \frac{B}{Z}$ at the other edge.

We have already established methods of finding the values in these formulae with exception of B and Z.

The value of B is found in the ordinary manner in accordance with established rules of mechanics and therefore need not be further explained in this article.

The value of Z however is a little more involved and necessitates the determination of the inertia (I) of the section ;

because $Z = \frac{2I}{d}$ (Assuming the section is symmetrical about its axis which is usually the case in this class of work).

The inertia is found as for a section composed of a homogeneous material such as steel or iron, with the exception that we make allowance for the area of the vertical reinforcement being $(m-1)$ times that for the concrete immediately surrounding it.

Therefore referring to Fig. 6, the inertia for the square section will be

$$I = \frac{bd^3}{12} + (m-1)\, A_v\, h_s^2$$

(neglecting the inertia of the reinforcement about its own axis, which would be very small),

and, $Z = \frac{2I}{d} = \frac{2}{d}\left\{\frac{bd^3}{12} + (m-1)A_v h_s^2\right\}$ or inserting bdr_v for A_v

$$Z = \frac{2}{d}\left(\frac{bd^3}{12} + (m-1)\, bdr_v h_s^2\right) = 2b\left(\frac{d^2}{12} + (m-1)\, r_v h_s^2\right)$$

In practice the difference between h_s and $\frac{d}{2}$ is often neglected,

which simplifies the formula somewhat, although it is not strictly correct.

For the oblong section, shewn in Fig. 6, the values for I and Z about the axis Y Y will be found in a similar manner to those for the square section, but about the axis X X only two-thirds of the reinforcement will be taken into account, as that falling on the axis will not give any increase of inertia, thus the values for I and Z in this case will be :—

$$I = \left(\frac{bd^3}{12} + \frac{2\,(m-1)A_v h_s^2}{3}\right) = \left(\frac{bd^3}{12} + \frac{2\,(m-1)\, bdr_v h_s^2}{3}\right) =$$

$$bd\left(\frac{d^2}{12} + \frac{2\,(m-1)\, r_v h_s^2}{3}\right) \text{ and } Z = \frac{2}{d}\left(\frac{bd^3}{12} + \frac{2\,(m-1)A_v h_s^2}{3}\right) =$$

$$\frac{2}{d}\left(\frac{bd^3}{12} + \frac{2\,(m-1)\, bdr_v h_s^2}{3}\right) = 2b\left(\frac{d^2}{12} + \frac{2\,(m-1)\, r_v h_s^2}{3}\right)$$

Following similar reasoning, the values of I and Z for the hexagonal section of column about axis X X will be

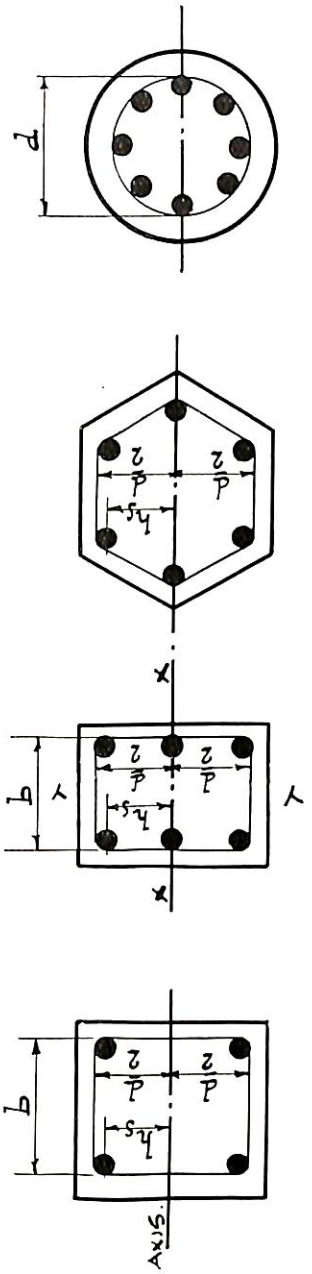


FIG 6.

$$I = \left(0{\cdot}12d^4 + \frac{2\,(m-1)A_v h_s^2}{3} \right) \text{ and}$$

$$Z = \frac{2}{d}\left(0{\cdot}12d^4 + \frac{2\,(m-1)A_v h_s^2}{3} \right)$$

and for the circular column,

$$I = \left(\frac{\pi d^4}{64} + \frac{(m-1)A_v d^2}{8} \right) = \left(\frac{Ad^2}{16} + \frac{(m-1)\,Ar_v d^2}{8} \right) =$$

$$\frac{Ad^2}{16}\left(1 + 2\,(m-1)\,r_v \right) \text{ and } Z = \left(\frac{\pi d^3}{32} + \frac{(m-1)A_v d}{4} \right) =$$

$$\left(\frac{Ad}{8} + \frac{(m-1)\,Ar_v d}{4} \right) = \frac{Ad}{8}\left(1 + 2\,(m-1)r_v \right)$$

Note the values for the circular column are nearly, but not quite correct, they are based on the assumption that the reinforcement forms a complete circle. To treat each bar of the reinforcement would give a tedious and very involved calculation.

Example—To design a column to withstand a direct load of 80,000 lbs., and a bending moment of 200,000 inch lbs. about its major axis (X X). It is first necessary to assume a section of column and then check that same meets the requirements. Assuming an oblong section, 10 in. by 15 in. effective area with 3% of vertical reinforcement in six bars and with $1\frac{1}{2}$% of lateral binding at $0{\cdot}2d$ pitch, the mixture of concrete being 1 : 2 : 4.

A = 10-in. × 15-in. = 150 square inches. Permissible value of c = 600 lbs. per square inch. A_v = 3% of 150 = 4·5 square inches, say six bars 1-in. diameter.

Permissible value of $i = c\,(1 + fsV_1)$ = 600 (1 + 16 × 0·015) = 744 lbs. per square inch.

$$m = \frac{9000}{i} = \frac{9000}{744} = \text{say } 12. \quad f = \frac{P}{A\,[1 + (m-1)\,r_v]} + \frac{B}{Z}$$

$$= \frac{P}{A\,[1 + (m-1)\,r_v]} + \frac{B}{2b\left\{ \frac{d^2}{12} + \frac{2\,(m-1)\,r_v h_s^2}{3} \right\}} =$$

$$\frac{80{,}000}{150\,[1 + 11 \times 0{\cdot}03]} + \frac{200{,}000}{2 \times 10\left\{ \frac{15 \times 15}{12} + \frac{2 \times 11 \times 0{\cdot}03 \times 7 \times 7}{3} \right\}}$$

= say 400 + or − 340.

Therefore the fibre stress on the concrete at the edge of maximum compression would be 400 + 340 = 740 lb. per square inch, which would be suitable as it does not exceed the permissible value of i, and the stress at the opposite edge would be 400 − 340 = 60 lb. per square inch, which would be quite allowable as no tension would be developed. The size of the binding would be found as previously described. It is usual to design columns so that no tension would be developed; if this is inconvenient at any time, care must be taken that there is enough steel to accommodate the tension, and the column would be more easily designed if treated as a beam with a longitudinal thrust, but this is a case which seldom occurs in actual practice.

Care should also be taken that excessive tension does not occur through the bending stresses being set up when the direct load is not at its maximum.

For instance, supposing the column we had in our example was one of the columns of a warehouse type of building and that the bending moment was caused by wind pressures. The wind pressure would be likely to occur when the warehouse was empty and the direct load was merely from the dead weight of the building. In this case the direct stress may be as low as say 100 lb. per square inch instead of 400 as worked out, therefore there would be considerable tension along one edge of the column, which would have to be accommodated by the vertical reinforcement, the column being treated as a beam as described later.

An alternative method of treating a column would be to fix the size of the column and vertical reinforcement and to find the fibre stress in the manner in which we found it in our example and then to provide lateral binding to suit the requirements of the case. This is perhaps a quicker method than the one adopted in our example.

LONG PILLARS AND COLUMNS.

Pillars of any description are subject to a buckling or crippling tendency in addition to the direct crushing stress.

This crippling does not assert itself until a certain proportion of length to width occurs, depending upon :—

(1) The materials of which the column is composed.
(2) The shape or section of the column.
(3) The method of securing the ends of the column.

In designing reinforced concrete columns we use an equivalent or virtual length (v) in accordance with the conditions of the ends, and design our column for a virtual length.

The following is a table shewing the values of v for varying conditions of ends as allowed by the L.C.C. Regulations.

Condition of Axis at Ends.	Virtual length v.
1. Both ends fixed in position and direction,	$v = l$
2. One end fixed in position and direction and one end fixed in position but not in direction,	$v = 1{\cdot}4l$
3. Both ends fixed in position but not in direction	$v = 2l$
4. One end fixed in position and direction and one end not fixed in position or direction,	$v = 4l$
	(l = length of axis)

Cases 1 and 2 are those most frequently met with in this method of construction. The values are rather stringent, but the question of the relative strength of columns with varying conditions of ends is probably the most debated question and causes the most controversy of any topic in structural engineering.

Having determined the virtual length of a column we proceed to design it, not for the actual load (W) which it supports, but for a value of P in excess of the load, according to the ratio of the virtual length to the width or to the radius of gyration.

The following is a table of values for W for varying ratio of $\frac{v}{d}$ and $\frac{v}{g}$ as allowed by the L.C.C. Regulations, intermediate values being in proportion to those tabulated.

Ratio of virtual length to effective width for rectangular pillars, ...	$\frac{v}{d}$ =	15	18	21	24	27	30
Ratio of virtual length to effective width for circular or hexagonal pillars	$\frac{v}{d}$ =	12	15	18	21	24	27
Ratio of virtual length to radius of gyration for any section of pillar, ...	$\frac{v}{g}$ =	45	54	63	72	81	90
Permissible load on column	W =	P	0·8P	0·6P	0·4P	0·2P	Nil.

It will be seen that a pillar must be treated as a long pillar when its virtual length exceeds 15 times its effective width in the case of a rectangular pillar, or 12 times its effective width in the case of circular pillars, or 45 times its radius of gyration. When the length is below these limits it is treated as a short pillar.

The values for $\frac{v}{d}$ appear to have been worked from columns with an average amount of reinforcement and higher proportions of P are often obtained if we work from the radius of gyration instead of the effective width.

The radius of gyration is found from the formula :—

$$g = \sqrt{\frac{I}{A + (m-1)\,A_v}} = \sqrt{\frac{I}{A\,[1 + (m-1)\,r_v]}}$$

Example : To find the permissible load (W) for a round pillar 10 feet long with an effective diam. of 10 inches, 2% of vertical reinforcement, and 1% of helical binding at $0{\cdot}2d$ pitch, assuming that one end is fixed in position and direction, and the other end in position only. The mixture of the concrete being—

1 : 1 : 2. $c = 750$ $i = 750\,(1 + 32 \times {\cdot}01)$ = say 1000 lbs. per sq. in.

$v = 1{\cdot}4l$, $= 1{\cdot}4 \times 120$, $= 168$ inches. $m = \frac{9000}{i} = 9$.

First, working from the effective diameter,

$$\frac{v}{d} = \frac{168}{10} = 16{\cdot}8 \text{ and,}$$

referring to the table the value for W for this ratio of $\frac{v}{d}$ for a circular column would be about 0·68 P.

Now, working from the radius of gyration

$$g = \sqrt{\frac{I}{A\,[1 + (m-1)\,r_v]}}$$

$$= \sqrt{\frac{\frac{Ad^2}{16}\,[1 + 2\,(m-1)\,r_v]}{A\,[1 + (m-1)\,r_v]}} = \sqrt{\frac{\frac{d^2}{16}\,[1 + 2\,(m-1)\,r_v]}{1 + (m-1)\,r_v}}$$

$$= \sqrt{\frac{\frac{10 \times 10}{16}\,[1 + 2 \times 8 \times {\cdot}02]}{1 + 8 \times {\cdot}02}} = \sqrt{\frac{8{\cdot}25}{1{\cdot}16}} = 2{\cdot}66.$$

$$\therefore \quad \frac{v}{g} = \frac{168}{2{\cdot}66} = 63$$

and referring to our tables for the value of W we find this is 0·6 P which is less than if we work from the values for $\frac{v}{d}$ This is due to the high value of i and the corresponding low value of m.

The value of P would be found in the manner previously described for short columns.

One might ask whether all these involved formulae have to be worked out for every reinforced concrete pillar that is designed ?

Certainly not ! By the aid of a few graphs it is surprising what a lot of calculation may be eliminated.

Most firms which specialise in this class of work have their own standard sections and a few of the more enterprising even issue handbooks giving their standard units of construction, complete with all properties of sections ready worked out. In any case the formulae for reinforced concrete work are no more involved than those for steel constructional work. We would be in a hopeless fix if we had to start working the section modulus or the radius of gyration for every section we used in that class of work, yet it is necessary to know how and why to do it, before we can thoroughly understand the application in practice.

The bases or foundations for reinforced concrete columns are dependent upon reasoning and formulae as applied to beams, and therefore we shall treat them later.

Before leaving columns there is just one other case which ought to be mentioned. Sometimes it is necessary to reduce the section of a column to an absolute minimum size on account of limited floor space or passage clearance and in order to obtain the necessary minimum of section a very high proportion of reinforcement has to be introduced. In a case of this description it is quite permissible to neglect the concrete altogether and to stress the reinforcement up to its full working limit of 16,000 lb. per square inch, neglecting the value of m altogether. This is equivalent to providing a steel column, the concrete merely acting as a cover and binding the steel together.

FLEXURE OF BEAMS.

Generally speaking, there are two ways in which a beam may fail, (1) by bending ; (2) by shearing. There are other ways, such as by torsion or by lateral weakness, which is really lateral bending.

First let us consider the bending or flexure of beams.

Bending Moments.—The bending moments as applied to reinforced concrete beams are determined by the ordinary static principles, but where beams are continuous over their supports, or where they have fixed ends, the negative bending moments must be taken into account. The beams must not be assumed to have free ends at these points, as is often done in the design of structural steelwork, otherwise the upper edge of the concrete at the points of negative bending will crack, due to the lack of tensile reinforcement.

(It will be observed that the upper edge is the tensile edge where a negative bending moment occurs). At the same time it must be kept in mind that when calculating bending moments for continuous beams, by the application of the theorem of three moments, it is assumed that all the points of support are level. Therefore if one of the supports sinks slightly, as in the case of a pillar slightly sinking due to earth settlement, the negative bending moment over this support is immediately reduced, with a corresponding increase in the adjacent positive bending moments. For this reason it is usual to increase the value of the positive bending moments for continuous beams to allow for any slight subsidence.

The following is a table of maximum bending moments due to variations in the incidence of distributed loads over approximately equal spans as recommended by the L.C.C. for continuous beams.

Near Middle of End Span.	At Support next to End Support.	At the Middle of Interior Spans.	At Other Interior Supports.
$+\frac{W_s l}{10}+\frac{W_d l}{12}$	$-\frac{W_s l}{9}-\frac{W_d l}{10}$	$+\frac{W_s l}{12}+\frac{W_d l}{24}$	$-\frac{W_s l}{9}-\frac{W_d l}{12}$
or approximately	or approximately	or approximately	or approximately
$+\frac{Wl}{10}$	$-\frac{Wl}{10}$	$+\frac{Wl}{12}$	$-\frac{Wl}{10}$

Where l = length of effective span measured centre to centre of support.

W = total weight or load (superimposed load + dead load) per span.

W_d = total dead load per span.

W_s = total superimposed load per span.

Although the L.C.C. recommend the foregoing table, they do not bind us to accept same, but allow the beams to be designed for the exact bending moments which will occur at any cross section of the beam, whether all the spans be loaded or any be unloaded. But even if this is done it is advisable to slightly increase, say by about 10%, the values obtained for the positive bending moments for the reasons previously stated.

A list is also given of bending moments allowed by the L.C.C. Regulations for beams of single span with varying conditions of ends.

Type of Load.	Ends.	Bending Moment (B) to be provided for.
Concentrated at centre of beam,	both free.	$B = + \frac{Wl}{4}$ at centre.
Uniformly distributed,	do.	$B = + \frac{Wl}{8}$ do.
Concentrated at centre,	both fixed.	$B = + \frac{Wl}{8}$ do., $-\frac{Wl}{8}$ at ends.
Uniformly distributed,	do.	$B = + \frac{Wl}{12}$ do., $-\frac{Wl}{12}$ at ends.
Uniformly distributed,	one fixed and one free.	$B = + \frac{Wl}{14}$ at $\frac{3}{8}$ of span from free end.
do. do.	one fixed and one free.	$B = - \frac{Wl}{8}$ at fixed end.
Concentrated at free end,	Cantilever	$B = - Wl$ at fixed end.
Uniformly distributed,	do.	$B = - \frac{Wl}{2}$ do.

It will be observed that for a beam, with both ends fixed, carrying a uniformly distributed load, the positive bending moment at the centre of the span is double that obtained by ordinary static methods, whilst for a similar beam carrying a concentrated load at the centre no corresponding increase is made, and the author strongly recommends that in this case the value of B should be $+ \frac{Wl}{7}$ at the centre of the beam.

In all our examples we will, however, adhere to the values of B as set forth by the L.C.C. to simplify our calculations.

MOMENT OF RESISTANCE OF SINGLE REINFORCED RECTANGULAR BEAMS.

Notation.

A = Total area of tensile reinforcement in square inches.
a = Arm of resistance moment, in inches.
B = Bending moment of the external forces in inch pounds.
b = Breadth of beam in inches.
c = The compressive fibre stress in the concrete, at the extreme edge of the beam, in lbs. per square inch.
d = Effective depth of the beam in inches, *i.e.* the distance from the compressed edge of the concrete to the centre of the area of the tensile reinforcement.
l = Length of effective span of beam in inches.
m = Modular ratio of elasticity.

n = Distance of the neutral axis from the compressed edge of the concrete.

n_1 = Ratio of n to d $= \frac{n}{d}$ Therefore $n_1 d = n$.

Q = qualifier in the equation $R = Qbd^2$

R = Moment of resistance.

r = Ratio of area of tensile reinforcement to effective area of the beam,

$$\text{i.e., } r = \frac{A}{bd} \text{ and } A = bdr.$$

t = The stress in pounds per square inch in the tensile reinforcement.

When a bending moment is applied to a beam, deflection is caused which sets up internal forces of resistance in the beam, thus where a beam is subject to a positive bending moment the upper edge of the beam is in compression, and the lower edge in tension, and *vice versa* for a negative bending moment. These internal forces acting in opposite directions about their centres of gravity form a moment of their own which is known as the moment of resistance and must safely withstand the bending moment without undue stress on the material either in tension or compression.

Fig. 7 shews a typical section and part elevation of a rectangular reinforced concrete beam with tensile reinforcement only (the shear reinforcement is omitted).

A strain diagram is also shewn, which can also be taken for a stress diagram because the stress is assumed to be proportional to the strain, but we must bear in mind that the stress in the reinforcement is in m times proportion to the compressive stress in the concrete.

It will be noticed that the stress changes over from a compression stress c at the upper edge to a tensile stress in the steel reinforcement. The neutral axis, or point where there is no stress, is not half way down the beam as would be the case in a beam composed of a homogeneous material such as steel, but is dependent upon the unit stresses c and t. The concrete below the neutral axis, which would be in tension, is neglected entirely so far as the moment of resistance is concerned, and therefore cannot govern the position of the neutral axis.

The compression is represented by the etched triangle above the neutral axis and the total compression will be

$\frac{cnb}{2}$ and will act at its centre of gravity which will be $\frac{n}{3}$ from

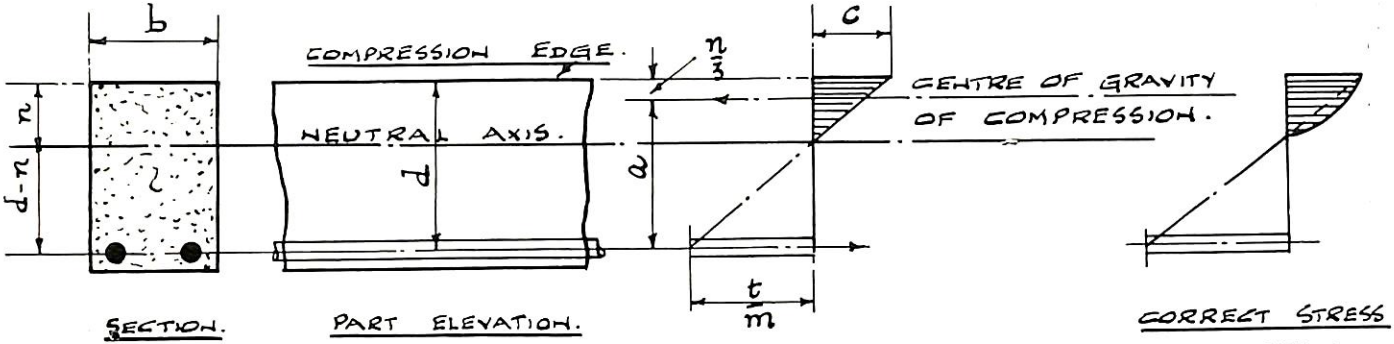

b
n
d-n
COMPRESSION EDGE.
NEUTRAL AXIS.
d
n/3
a
c
CENTRE OF GRAVITY
OF COMPRESSION.
t/m
SECTION.
PART ELEVATION.
ASSUMED STRESS DIAGRAM
CORRECT STRESS
DIAGRAM.

FIG 7.

the top or compressive edge of the concrete. The moment of resistance will be

$$R = \frac{bcn}{2} \times a, = \frac{bcn}{2} \times \left(d - \frac{n}{3}\right)$$

$$= \frac{bcdn_1}{2} \times \left(d - \frac{dn_1}{3}\right), = \frac{bd^2\, cn_1}{2}\left(1 - \frac{n_1}{3}\right)$$

Strictly speaking, these values are on the low side because the stress in the concrete is not proportional to the strain and therefore the compression triangle should assume the parabolic shape shewn on the extreme right of the diagram, but the benefit derived from introducing the parabola into our calculations would be very small and considerably add to the labour in working out the formulae ; further, its introduction is not allowed by the authorities.

Now, working from the values of the tensile stresses. Total tension = At and the moment of Resistance (R) =

$$At \times a = bdrt \times \left(d - \frac{n}{3}\right) = bdrt \times \left(d - \frac{n_1 d}{3}\right) = bd^2rt\left(1 - \frac{n_1}{3}\right)$$

As the value of R must be the same when worked either from the values of the concrete or the steel, we can equate the two values and we get—

$$R = \frac{bd^2\, cn_1}{2}\left(1 - \frac{n_1}{3}\right) = bd^2rt\left(1 - \frac{n_1}{3}\right)$$

and as bd^2 is a common factor, the formula could be written

$$R = bd^2\, Q \text{ where } Q = \frac{cn_1}{2}\left(1 - \frac{n_1}{3}\right) \text{ or } Q = rt\left(1 - \frac{n_1}{3}\right)$$

It is easily observed that the value of Q is dependent upon the value of n_1, to a large extent. Equating the values of Q we get :—

$$\frac{cn_1}{2}\left(1 - \frac{n_1}{3}\right) = tr\left(1 - \frac{n_1}{3}\right) \text{ or } \frac{cn_1}{2} = tr$$

from which we get :—

$$c = \frac{2\,tr}{n_1} \text{ and } t = \frac{cn_1}{2r} \text{ and } r = \frac{cn_1}{2t} \text{ and } n_1 = \frac{2\,tr}{c} \text{ and } \frac{c}{t} = \frac{2r}{n_1}$$

Now resolving the triangles in Fig. 7,

$$c : n :: \frac{t}{m} : d - n \text{ or } \frac{c}{n} = \frac{t}{m\,(d-n)} \text{ or } \frac{c}{n_1 d} = \frac{t}{md\,(1-n_1)} \text{ and}$$

$$\frac{c}{t} = \frac{n_1}{m\,(1-n_1)}$$

Equating the two values for $\frac{c}{t}$ we get $\frac{n_1}{m\,(1-n_1)} = \frac{2r}{n_1}$

$$\therefore \quad n_1^2 = 2rm\,(1-n_1) = 2rm - 2rmn_1$$
$$\therefore \quad n_1^2 + 2rmn_1 = 2rm$$

which may be worked as a quadratic equation and by adding $r^2\,m^2$ to each side of the equation we get

$$n_1^2 + 2rmn_1 + r^2\,m^2 = 2rm + r^2m^2$$

and finding the sq. root of these we get

$$n_1 + rm = \sqrt{2rm + r^2\,m^2} \qquad \therefore n_1 = \sqrt{2rm + r^2\,m^2} - rm$$

and as r is a known quantity for any given beam and m depends entirely upon the mixture of the concrete, we are thus able to obtain the value of n_1 for any given rectangular beam with single reinforcement.

Example :—To find the position of the neutral axis of a beam with 0·7% of tensile reinforcement, the mixture of the concrete being $1 : 1\frac{2}{3} : 3\frac{1}{3}$ the effective depth of the beam being 20 inches.

$$m \text{ for this mixture of concrete} = \frac{9000}{650} = \text{say } 13{\cdot}85$$

$$n_1 = \sqrt{2rm + r^2\,m^2} - rm$$
$$n_1 = \sqrt{2 \times 0{\cdot}007 \times 13{\cdot}85 + 0{\cdot}007^2 \times 13{\cdot}85^2} - 0{\cdot}007 \times 13{\cdot}85$$
$$n_1 = \sqrt{0{\cdot}1939 + 0{\cdot}00939} - 0{\cdot}09695 = \text{say } 0{\cdot}353$$

Therefore the position of the neutral axis would be 20-in. $\times 0{\cdot}353$ = 7·06 inches from the compression edge of the beam.

Figure 8 is a chart giving the values for n_1 for rectangular beams, with tensile reinforcement only, for varying mixtures of concrete and proportions of reinforcement.

Having found the value of n_1 we are able to find the stress c or t for any given beam subject to a bending moment, bearing in mind that c and t are the actual stresses and not necessarily the allowable stresses in their respective materials.

Example :—To find the stresses c and t for a rectangular beam 6-in. wide, effective depth 10-in., reinforced with 1% of tensile reinforcement, the mixture being 1 : 6, subject to a bending moment of 70,675 inch pounds.

From either the formula or the chart we find n_1 equals about 0·414. To find the stress t :—

$$bd^2\,tr\,(1 - \tfrac{n_1}{3}) = 70{,}675 \qquad \therefore t = \frac{70{,}675}{bd^2\,r\,(1 - \frac{n_1}{3})} =$$

$$\frac{70{,}675}{6 \times 100 \times 0{\cdot}01 \times 0{\cdot}862} = \text{say } 13{,}650 \text{ lbs. per sq. inch.}$$

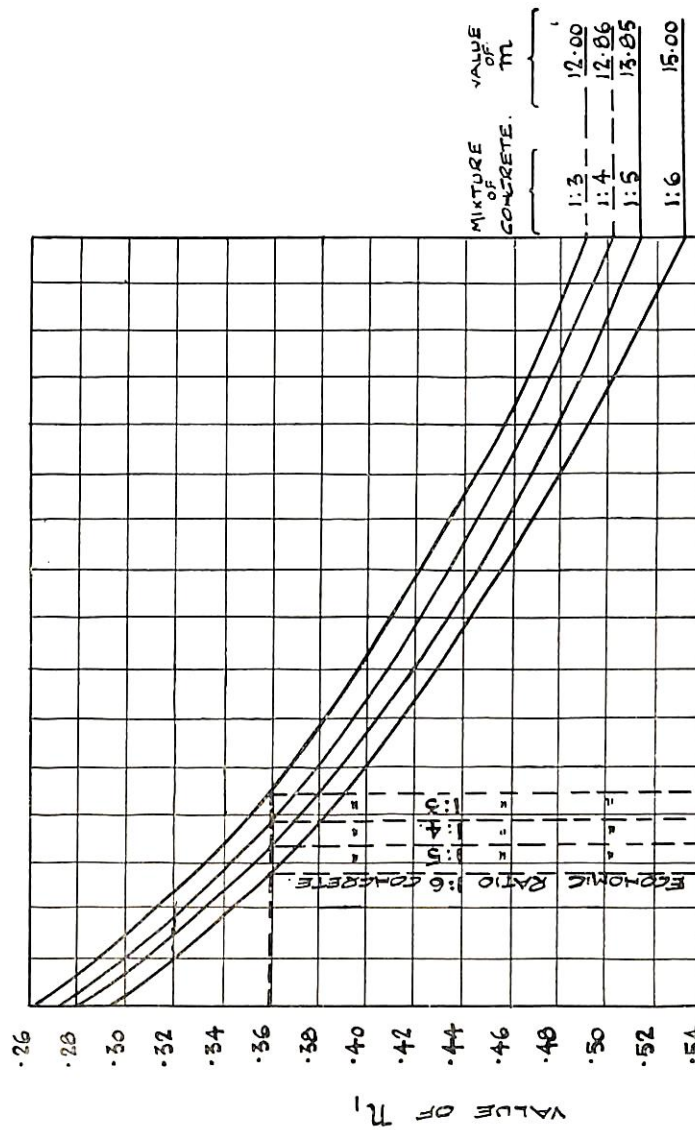

VALUE OF n_1
·26
·28
·30
·32
·34
·36
·38
·40
·42
·44
·46
·48
·50
·52
·54
0·4 ·5 ·6 ·7 ·8 ·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
PERCENTAGE OF REINFORCEMENT.
ECONOMIC RATIO 1:6 CONCRETE
·615
·731
·788
·844
1:3
1:4
1:5
1:6
MIXTURE OF CONCRETE.
VALUE OF m
12·00
12·86
13·85
15·00

FIG. 8.

To find the stress c :—

$$\frac{bd^2 cn_1}{2}\left(1 - \frac{n_1}{3}\right) = 70{,}675 \therefore c = \frac{70{,}675 \times 2}{bd^2 n_1 (1 - \frac{n_1}{3})} =$$

$$\frac{70{,}675 \times 2}{6 \times 100 \times 0{\cdot}414 \times 0{\cdot}862} = \text{say } 660 \text{ lbs. per sq. inch.}$$

It is obvious the beam is badly designed, because though the stress in the reinforcement is well below the working limits, the stress in the concrete is above the allowable working stress. This example has purposely been selected to shew that either the value of c or t may be safe without it necessarily following that both are safe.

Therefore if we reverse the order of working, and work from the formulae,

R = bd^2 Q and find Q from the formulae previously given, *i.e.*,

$$Q = \frac{cn_1}{2}\left(1 - \frac{n_1}{3}\right) \text{ or } Q = tr\left(1 - \frac{n_1}{3}\right)$$

and using the working limits of c and t to find the moment of resistance of the beam, we must work from the lesser of the two values of Q.

Example :—To find the moment of resistance of the beam in the previous example.

Working from the compressive value :—

$$Q = \frac{cn_1}{2}\left(1 - \frac{n_1}{3}\right) = \frac{600 \times 0{\cdot}414}{2} \times 0{\cdot}862 = \text{say } 107$$

and from the tensile value :—

$$Q = tr\left(1 - \frac{n_1}{3}\right) = 16{,}000 \times 0{\cdot}01 \times 0{\cdot}862 = \text{say } 138$$

It is obvious that the beam is weaker in compression than in tension, therefore we must use the value of Q as found from compression

$\therefore$ R = bd^2 Q = 6 × 100 × 107 = 64,200 inch lbs.,

which is the bending moment the beam would safely resist.

It is not necessary, however, to find both the values of Q, as will be seen later.

Economic Ratio.

A beam is said to be in economic ratio when the values of c and t are the safe working stress their respective materials will safely resist, and it is with this object in view that a reinforced concrete beam is usually designed.

To find this ratio we must find the value of r and equate it with the stresses the materials will resist.

We have already established that :—

$r = \frac{cn_1}{2t}$ but as n_1 is dependent upon r we must substitute another value for n_1.

We have proved that :—

$$\frac{c}{t} = \frac{n_1}{m\,(1 - n_1)}$$

$$\therefore cm - cmn_1 = n_1 t \quad \therefore cm = n_1 t + cmn_1 = n_1 (t + cm)$$

$$\therefore n_1 = \frac{cm}{t + cm}$$

and substituting this value of n_1 in the formula $r = \frac{cn_1}{2t}$ we get

$$r = \frac{c^2 m}{2t\,(t + cm)}$$

Example :—To find the economic ratio and the values of n_1 and Q for any rectangular reinforced concrete beam, with tensile reinforcement only, for a 1 : 6 mixture of concrete, using L.C.C. values of c and t.

$$r = \frac{c^2 m}{2t\,(t + cm)} = \frac{600 \times 600 \times 15}{2 \times 16{,}000\,(16{,}000 + 600 \times 15)}$$

$$= 0{\cdot}00675 \text{ or } 0{\cdot}675\%$$

and from either the Table in Fig. 8 or from the formula :—

$$n_1 = \sqrt{2rm + r^2 m^2} - rm$$

$$= \sqrt{2 \times 0{\cdot}00675 \times 15 + 0{\cdot}00675^2 \times 15^2} - 0{\cdot}00675 \times 15 = 0{\cdot}36$$

$$\text{and } Q = tr\left(1 - \frac{n_1}{3}\right) \text{ or } \frac{cn_1}{2}\left(1 - \frac{n_1}{3}\right)$$

$$= 16{,}000 \times 0{\cdot}00675 \times 0{\cdot}88 \text{ or } \frac{600 \times 0{\cdot}36}{2} \times 0{\cdot}88 = 95{\cdot}04$$

in both cases, and for ease in calculation it is usual to assume this value as 100 for a 1 : 6 mixture of concrete.

The following is a list of values for rectangular beams with tensile reinforcement only, when in economic ratio, as computed by the foregoing formulae and in accordance with the L.C.C. regulations. The economic values of r and n_1 are also shewn on Fig. 8 by dotted ordinates.

BEAMS IN ECONOMIC RATIO TO L.C.C. REQUIREMENTS.

Mixture	c	t	m	r	n_1	Correct value of Q.	Assumed value of Q to simplify calculations.
1 : 6	600	16,000	15·00	0·00675	0·36	95	100
1 : 5	650	,,	13·85	0·00731	0·36	103	100
1 : 4	700	,,	12·86	0·00788	0·36	111	110
1 : 3	750	,,	12·00	0·00844	0·36	119	120

Example :—To find the required cross section of a rectangular beam in economic ratio, 15-ft. 0-in. span with both ends fixed, to carry an evenly distributed load of 20 tons, the mixture of the concrete being 1 : 4. Q = 110

$$B = + \frac{W\,l}{12} \text{ at centre and } - \frac{W\,l}{12} \text{ at ends.}$$

$$\therefore\ B = \frac{20 \times 2{,}240 \times 180}{12} = 672{,}000 \text{ inch lbs.}$$

Equating the moment of resistance and the bending moment we get :

$$bd^2\,Q = 672{,}000 \text{ and } bd^2 = \frac{672{,}000}{Q} = \frac{672{,}000}{110}$$

We may either assume a value for b and find the value of d or *vice versa*, or we may assume b to be in a certain ratio with d.

When there is nothing to limit either b or d a good proportion is for b to be about $0{\cdot}6d$ and assuming this ratio in our example, we get :—

$$0{\cdot}6d^3 = \frac{672{,}000}{110} \text{ and } d = \text{say } 21{\cdot}75 \text{ inches.}$$

$b = 0{\cdot}6d = 13$ inches.

A = 0·00788 × 13 in. × 21·75 ins. = 2·23 sq. in.,

say six bars 11/16 in. diameter, which would be placed near the underside of the beam at the centre and near the top at the ends. The bars could be stopped as the bending moment diminishes towards the point of contra-flexure (in a similar manner to which the flanges of a steel plate girder are reduced) and hooked into the concrete or they may be inclined across the beam in continuous units.

Care must be taken if the bars are stopped that sufficient length is allowed beyond the theoretical stopping point of a bar to thoroughly develop its grip on the concrete. The L.C.C. regulations demand

that reinforcement (not necessarily the whole) must be carried beyond the points of contra-flexure, under any condition of loading, by a length at least equal to half the effective depth of the beam. As the point of contra-flexure is liable to considerable variation due to variations of loading, it is advisable to carry the reinforcement well beyond this point. The reinforcement bars may either be placed side by side, or in pairs of bars, the one over the other, the effective depth (d) being measured to their common centre of gravity, but sufficient cover of concrete must be allowed, which, to comply with the L.C.C. regulations, must be 1-in. or the diameter of the bar, whichever is the greater, also the bars must not be spaced too close together or the ramming of the concrete will be impeded. The L.C.C. require a minimum space between bars of 1-in. except at joints or crossings of same.

No tensile reinforcement should have a diameter of less than a quarter of an inch except in the case of mesh reinforcement for slabs, which should not be less than one-tenth of an inch in diameter.

MOMENT OF RESISTANCE OF T (TEE) BEAMS

(With Tensile Reinforcement Only).

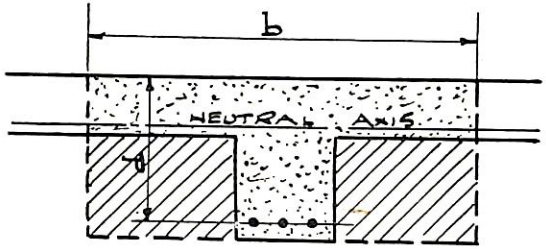
When a beam supports a slab such as a beam supporting a floor slab, a part of the slab is taken into account as being the compression portion of the beam. It is obvious that the whole width of the slab cannot be assumed to be part of the beam, and the following limits of the breadth of the slab allowed to be calculated with the beam are set forth by the L.C.C. regulations :—

" Not more than a quarter the effective span of the beam, nor more than twelve times the thickness of the slab, and not more than the distance between the centre of the ribs of the Tee beams, whichever is the least."

There are three cases of Tee beams :—Case 1, in which the neutral axis falls within the slab ; Case 2, in which the neutral axis coincides with the underside of the slab ; and Case 3, where the neutral axis is below the bottom of the slab. Typical cross sections of Cases 1 and 2 are shown in Fig. 9. In either of these cases the formulae are exactly similar to those for rectangular beams, with a breadth (b) of the width of slab assumed to be part of the beam, as though the etched portions had been cut away, being of no use as they would be in tension, and the tensile resistance of the concrete is neglected.

Case 3 is somewhat different, as will be seen on reference to the stress diagram in Fig. 10. The triangle of compression which we had in the case of a rectangular beam gives place to the quadrilateral of compression shewn etched in the stress diagram. The compression in the rib below the bottom of the slab is neglected to

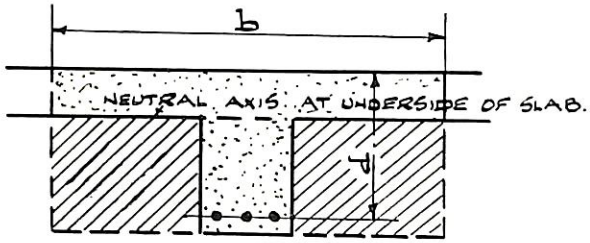
DEPTH OF NEUTRAL AXIS FOR RECTANGULAR BEAMS, SLABS & (T BEAMS CASES 1 AND 2) WHEN REINFORCED IN TENSION ONLY.



T BEAM. (CASE 1)

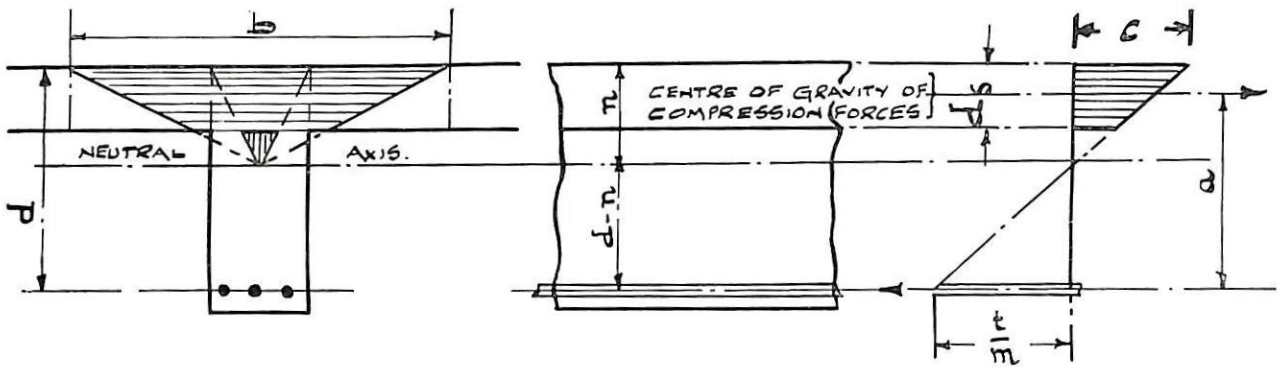
NEUTRAL AXIS WITHIN SLAB.

FIG. 9



T BEAM (CASE 2)

NEUTRAL AXIS AT UNDERSIDE OF SLAB.



CROSS SECTION (ETCHED PART INDICATES EQUIVALENT COMPRESSION AREA AND THE VERTICAL ETCHED PORTION THE NEGLECTED COMPRESSION)

PART ELEVATION

STRESS DIAGRAM.

T BEAM (CASE 3)

NEUTRAL AXIS OUTSIDE SLAB.

FIG. 10

simplify calculations, its value however is very small, as will be seen on reference to the equivalent compressional area etched in the cross section of the beam.

Notation.—The notation used for rectangular beams with the following additions is used for Tee beams, Case 3.

b_r = breadth of rib in inches.
d_s = depth of slab in inches.

$$s_1 = \text{slab depth ratio} = \frac{d_s}{d}$$

It is not necessary to give the working out of all the formulae for the resistance to bending for this class of beam, because the reasoning is similar to that for rectangular beams.

The mean compression in the quadrilateral of compression

$$= c - \frac{cd_s}{2n} = c\left(1 - \frac{d_s}{2n}\right) = c\left(1 - \frac{s_1 d}{2n_1 d}\right) = c\left(1 - \frac{s_1}{2n_1}\right)$$

Therefore the total compression

$$= bd_s\, c\left(1 - \frac{s_1}{2n_1}\right) = bds_1\, c\left(1 - \frac{s_1}{2n_1}\right)$$

The lever arm $= d - \frac{d_s}{3}\left(\frac{3n - 2d_s}{2n - d_s}\right) = d\left\{1 - \frac{s_1}{3}\left(\frac{3n_1 - 2s_1}{2n_1 - s_1}\right)\right\}$

The moment of resistance (R) from the compression values = Total compression × lever arm.

$$\therefore R = bds_1\, c\left(1 - \frac{s_1}{2n_1}\right) \times d\left\{1 - \frac{s_1}{3}\left(\frac{3n_1 - 2s_1}{2n_1 - s_1}\right)\right\} =$$

$$bd^2 s_1\, c\left(1 - \frac{s_1}{2n_1}\right) \times \left\{1 - \frac{s_1}{3}\left(\frac{3n_1 - 2s_1}{2n_1 - s_1}\right)\right\} = bd^2\, Q \text{ where } Q =$$

$$s_1\, c\left(1 - \frac{s_1}{2n_1}\right) \times \left\{1 - \frac{s_1}{3}\left(\frac{3n_1 - 2s_1}{2n_1 - s_1}\right)\right\}$$

The moment of resistance (R) from the tensile values = Total tension × lever arm

$$\therefore R = rbdt \times d\left\{1 - \frac{s_1}{3}\left(\frac{3n_1 - 2s_1}{2n_1 - s_1}\right)\right\}$$

$$= bd^2 tr\left\{1 - \frac{s_1}{3}\left(\frac{3n_1 - 2s_1}{2n_1 - s_1}\right)\right\}$$

$$= bd^2\, Q \text{ where } Q = tr\left\{1 - \frac{s_1}{3}\left(\frac{3n_1 - 2s_1}{2n_1 - s_1}\right)\right\}$$

By equating the values of Q we get $\frac{c}{t} = \frac{2rn_1}{(2n_1 - s_1)\, s_1}$

Resolving the triangle in the stress diagram we get

$\frac{c}{t} = \frac{n_1}{m\,(1 - n_1)}$ and equating the two values of $\frac{c}{t}$ we get

$$\frac{2rn_1}{(2n_1 - s_1)\, s_1} = \frac{n_1}{m\,(1 - n_1)} \text{ and } n_1 = \frac{s_1^2 + 2mr}{2\,(s_1 + mr)}$$

inserting this value for n_1, we find :—

$$Q = cs_1 \left(\frac{s_1^3 + 4mrs_1^2 - 12mrs_1 + 12mr}{6\,(s_1^2 + 2\,mr)} \right)$$

when working from the compressive values, and :—

$$Q = t \left(\frac{s_1^3 + 4mrs_1^2 - 12mrs_1 + 12mr}{6m\,(2 - s_1)} \right)$$

when working from the tensile values, the least of which must be used when determining the moment of resistance of the beam.

The economic value of r from similar reasoning to that used for the rectangular beam will be found from the formula :—

$$r = \frac{2mcs_1 - mcs_1^2 - ts_1^2}{2mt}$$

Values for n_1, r and Q can be plotted from the foregoing formulae for Tee beams with any value of s_1 for a given mixture of concrete, but space will not permit of them being included in this article.

The application of the formulae is similar to the application for rectangular beams and therefore it is hardly necessary to work out an example, but it should be pointed out that where the negative bending moments occur the breadth of the rib will become the breadth of the beam, because the underside is the part of the beam in compression at these points and the beam becomes a rectangular beam and is treated as such. It will, however, be seen that the value of the compression area of the beam at the point of negative bending will have to be increased above the ordinary area of the rib on account of the reduction in breadth.

There are four ways of making this increase :—

(1) By increasing the depth of the beam.
(2) By reinforcing the compression area with steel bars.
(3) By hooping or binding the concrete in compression, and thus increasing the permissible stress in the concrete in a similar manner to the method adopted for a pillar with hooped core.
(4) By a combination of any of the three previous methods.

It should be stated that when the depth of the beam is increased, which amounts to putting a splayed bracket to the underside of the beam, such splayed work should not be calculated at more than 30 degrees to the horizontal, because this is about the angle of shear dispersion of concrete.

When compressive reinforcement is introduced the beam is treated as a double reinforced beam to be described later.

RESISTANCE OF SLABS.

The resistance of a slab is calculated in a similar manner to that for a rectangular beam of unit breadth (usually 1 foot is assumed for calculation purposes).

But when the slab is supported on more than two edges, as shewn in Fig. 11, the slab is resisting bending in both directions and the bending moment to be allowed for in each direction causes no little amount of controversy amongst engineers.

To overcome this difficulty some engineers reinforce the slab in one direction only, and neglect the support from the other edges. This method is strongly to be condemned, especially if the slab is fixed to, or continuous over the neglected supports, because bending moments are bound to occur in this direction and would probably crack the slab, although it would not fail altogether.

A much better method is to design the slab as though a portion of the load were carried in each direction.

The proportion of load to be carried in each direction would vary according to the length and breadth of the slab.

The L.C.C. regulations allow the following proportions of the load to be carried in each direction.

The load to be carried in the direction of the breadth of the slab

$$W_1 = \frac{W}{1 + \left(\frac{B}{L}\right)^4}$$

where W = total load on slab
L = length of slab
B = breadth of slab

and the load to be carried in the direction of the length of the slab

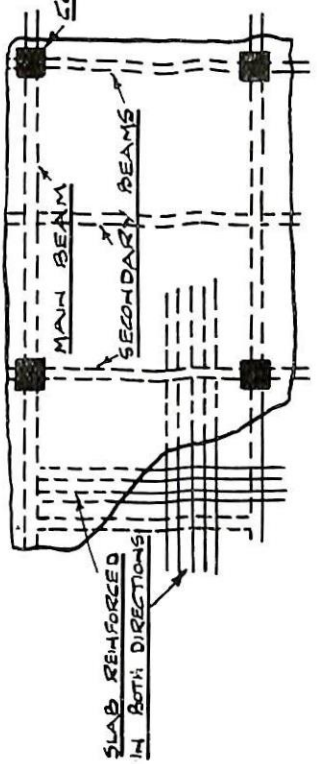
$$W_2 = \frac{W}{1 + \left(\frac{L}{B}\right)^4}$$

but when the length of the slab exceeds twice the breadth the whole of the load must be carried in the direction of the breadth of the slab.

Example :—To find the unit bending moment on a slab 16-ft. long, and 12-ft. wide freely supported along all edges, the load being 200 lb. per square foot.

FIG 11.

PART FLOOR PLAN SHEWING SLAB REINFORCEMENT.



The load per square foot to be carried in the direction of the width

$$W_1 = \frac{200}{1 + \left(\frac{12}{16}\right)^4} = \frac{200}{1 + 0{\cdot}32}$$

= say 150 lbs. per square foot and the bending moment would be $\frac{150 \times 12 \times 12 \times 12}{8}$ = 32,400 inch lbs. per foot width of slab.

The load per square foot to be carried in the direction of the length

$$W_2 = \frac{200}{1 + \left(\frac{16}{12}\right)^4} = \text{say } 50 \text{ lbs. per square foot.}$$

This second value may, of course, be found by deducting the proportion to be provided for in the breadth from the whole, *i.e.*, 200 — 150 = 50.

The bending moment in the direction of the length of the slab would be $\frac{50 \times 16 \times 16 \times 12}{8}$ = 19,200 inch lbs. per foot width.

The distance between the centres of the reinforcing bars should not be greater than 12 inches and not more than twice the effective depth of the slab.

In cases where the slab is reinforced in one direction only, distributing or spacing bars should be placed at right angles to the reinforcement, at not greater than 18-in. centres, nor more than four times the effective depth of the slab, the diameter of such bars should not be less than 1/16th of the effective depth of the slab, nor less than one-tenth of an inch.

A favourite method of distributing the reinforcement in a slab is to put half the reinforcement in the middle third of the slab in both directions. This considerably increases the strength of the slab, as undoubtedly the centre is the weakest part. When this is done a reduction may be made in the total amount of reinforcement, although the L.C.C. do not allow of such reduction.

There are many forms of steel meshings on the market which form admirable reinforcement for slabs. The cover of concrete for slab reinforcement should not be less than $\frac{1}{2}$-in. nor less than the diameter of the bar to be covered.

SELF-SUPPORTING SLABS.

This type of slab, whilst not being so economical in materials, gives a great saving in the centering or formwork, as the ribs or Tee beams are dispensed with and the centering may then be continuous under the whole of the slab.

The self-supporting slab is reinforced either diagonally or radially from the columns. The mushroom type of flooring is one of the self-supporting type.

⌐ (ELL) BEAMS.

Ell beams such as the boundary beams supporting the edge of a slab are designed in a similar manner to Tee beams, but the width (b) of the slab assumed as part of the beam is, of course, limited to a further degree than it is for a Tee beam.

The L.C.C. regulations allow for the breadth of slab to be assumed as the compression flange, to be taken at not more than four times the thickness of the slab, but provided that moments due to the eccentricity of the compression are provided for, this width may be increased, but not beyond one-half the width allowed for Tee beams.

BEAMS WITH DOUBLE REINFORCEMENT.

Very often on account of limited headroom, or in cases where a reversal of stress is likely to occur, it is found necessary to reinforce the compressive area of a beam. Compressive reinforcement is also often adopted at the points of negative bending in Tee beams in lieu of increasing the depth of the rib of the beam, thus obviating the necessity of splayed work, or at least reducing the splayed work to a minimum, which is a great saving in the form-work or centering.

When compressive reinforcement is used it should be effectively anchored into the concrete to prevent it from bursting the compression edge of the beam.

The L.C.C. regulations demand that compressive reinforcement be anchored not further apart than a distance equal to the lever arm of the forces in the beam, and not further apart than 16 times the diameter of the anchored bar, by bars extending through a depth at least equal to the lever arm.

In practice the compressive reinforcement is usually anchored by means of the shear reinforcement to be described later.

Figure 12 shews a cross section, part elevation and stress diagram of a rectangular beam with double reinforcement.

It is very seldom that the compression area of a Tee beam requires reinforcing, except at the points of negative bending where the beam resolves itself into an inverted rectangular beam with a breadth equal to the breadth of the rib.

Should a Tee beam require reinforcing, the same reasoning will apply as that which follows for rectangular beams, with the exception that the formulae for Tee beams will have to be used in lieu of those for rectangular beams.

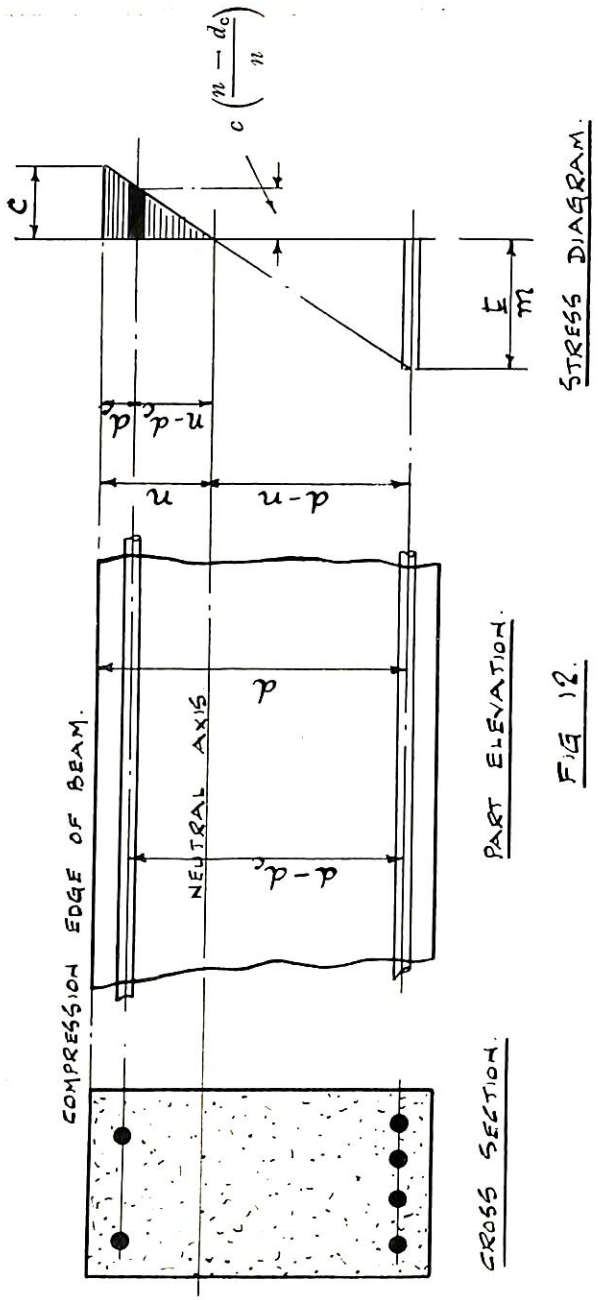

$c\left(\frac{n-d_c}{n}\right)$
c
STRESS DIAGRAM.
$\frac{f}{m}$
p
p-u
u
u-p
p
NEUTRAL AXIS
p-p
COMPRESSION EDGE OF BEAM.
PART ELEVATION.
CROSS SECTION.

FIG 12.

Notation.—The notation for beams with tensile reinforcement only will apply with the addition of

d_c = distance of centre of compressive reinforcement from the compressive edge of the concrete in inches.

A_c = total area of compressive reinforcement.

The stress in the compressive reinforcement will be m times that in the concrete immediately surrounding it, which will depend upon the distance the compressive reinforcement is from the neutral axis.

$\therefore$ The stress in the compressive reinforcement $= \dfrac{cm\ (n-d_c)}{n}$

But we must not lose sight of the fact that compressive reinforcement displaces its own area of concrete and therefore we may say that the value of the stress in the compressive reinforcement

$$= \frac{c\ (m-1)\ \times\ (n-d_c)}{n}$$

and the total value of the stress in the compressive reinforcement

$$= \frac{A_c\ c\ (m-1)\ \times\ (n-d_c)}{n}$$

and the moment of resistance of this value

$$= \frac{A_c\ c\ (m-1)\ \times\ (n-d_c)\ \times\ (d-d_c)}{n}$$

which may be added to the moment of resistance as found from the compressive values for a single reinfored concrete beam. The result or formula so obtained may be reduced considerably by introducing values for :—

$$\frac{d_c}{d} \text{ of } d_1 \text{ and for } A_c \text{ of } r_1\ bd$$

Also we notice that a fresh value for n will be found if we equate total tension and total compression, or the two values of R as found from the compression and tension. And as the lever arm for the steel in compression is not necessarily the lever arm for the concrete in compression, one will realise that considerable calculation is involved in solving the formulae for the moment of resistance of beams with double reinforcement and as these formulae are seldom used in practice it is not proposed to inlude them in this article, but to explain how a double reinforced concrete beam is usually treated in practice.

In the first place it is usually assumed that the centre of gravity of the compressive reinforcement coincides with that of the compressive stresses in the concrete, thus assuming one lever arm of

resistance. (If this assumption is far from being correct in any case, then both lever arms should be taken into account, but this is an abnormal case).

We next find the area of steel required for the tensile reinforcement from the formula for single reinforced beams assuming the values of n_1 and c are those for economic ratio. This will give a value for r in excess of the economic ratio for a single reinforced beam which must be compensated by compressive reinforcement.

Example :—To find the required area of tensile and compressive reinforcement for a reinforced concrete beam of 1 : 2 : 4 concrete, 10-in. broad and 15-in. effective depth. The bending moment being 320,000 inch lbs.

In this case t = 16,000, c = 600, m = 15, lever arm = 0·88d.

A required $= \dfrac{320{,}000}{16{,}000 \times 15 \times 0{\cdot}88} =$ 1·51 square ins.

The economic ratio for a single reinforced concrete beam of this mixture = 0·00675 and the corresponding area of steel would be 0·00675 × 10 × 15 = 1·01 square ins.

Excess to be compensated by compressive reinforcement = 0·5 square ins.

Value of stress in compressive reinforcement $= \dfrac{c\,(m-1) \times (n-d_c)}{n}$

and as we have assumed d_c to be $\dfrac{n}{3}$ the value of the stress in the compressive reinforcement $= \dfrac{600 \times 14 \times 2}{3} =$ 5,600 lbs. per sq. inch and the required area A_c of compressive reinforcement $= \dfrac{0{\cdot}5 \times 16{,}000}{5600} =$ say 1·43 sq. inch.

DOUBLE REINFORCED BEAMS WHEN CONCRETE IS NEGLECTED.

When the value of A_c, as found in the previous example, exceeds the value of A it is permissible to neglect the compression in the concrete and to develop the full working stress (*i.e.* 16,000 lbs. per sq. inch) in the steel. Sometimes to keep the same amount of reinforcement in compression and in tension this method is adopted. Care should be taken that the compressive reinforcement is more

efficiently anchored in this case and the L.C.C. regulations require the compressive reinforcement to be anchored both vertically and laterally by anchors not further apart than 6 inches, centre to centre, and not further apart than 8 times the diameter of the anchored bar, and the anchors should be passed round or hooked over both the compressive and the tensile reinforcement.

SHEAR STRESSES IN BEAMS.

The vertical and horizontal shear stresses in a reinforced concrete beam set up diagonal compression and tensile stresses in a very similar manner to those in a steel beam, and to be strictly correct it is the diagonal tension which we have to guard against when designing a reinforced concrete beam. The diagonal stresses from similar reasoning to that applied to the webs of steel plate girders would appear to be inclined at an angle of about 45 degrees to the horizontal. This is further verified by tests which have been made on concrete beams. When the beams have failed, due to shearing stresses, the failure has invariably been inclined across the beam in something like the manner shewn in Fig. 13.

FIG 13. SHEWING CRACKS IN BEAM DUE TO FAILURE BY DIAGONAL TENSION.

Considerable controversy exists regarding the portion of a beam which may be considered to resist shear.

Some engineers claim that the whole sectional area of the concrete in the rib of the beam (which in the case of a rectangular beam amounts to the whole area of the beam) resists shear and consider that the beam would be quite safe if the unit shear did not exceed the shearing resistance of the concrete, *i.e.*, one-tenth of the compressive resistance of the concrete.

Other engineers contend that only the depth of the lever arm of the beam should be considered to resist shear, because this is the vertical height or component from which the diagonal stresses are bound to originate. This method is generally accepted and is probably the more correct on account of the uneven distribution of shear stress over the sectional area of the beam. Further, the authorities generally adopt the latter method. Therefore the intensity

of the shearing stress for a concrete beam, or slab, without shear reinforcement should not exceed the foregoing values when calculated from the formula :—

$$s = \frac{S}{b_r\, a}$$

where s = shear stress in lbs. per sq. inch.
S = total shear in lbs.
b_r = breadth of rib of a tee beam or breadth of a rectangular beam in inches.
a = lever arm of beam in inches.

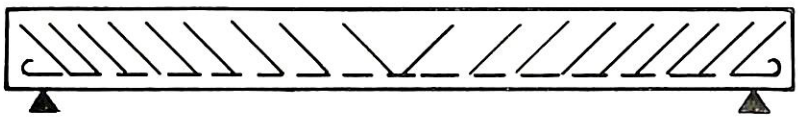


FIG 14. SHEWING ARRANGEMENT OF STIRRUPS TO PREVENT FAILURE BY DIAGONAL TENSION.

When s exceeds this value, shear reinforcement must be introduced, which usually takes the form of stirrup bars wrapped round or otherwise attached to the longitudinal reinforcement and inclined across the neutral plane of the beam at an angle of about 45 degrees, as shewn in Fig. 14, to prevent failure by diagonal tension. Fig. 15 shews various forms of stirrup arrangements.

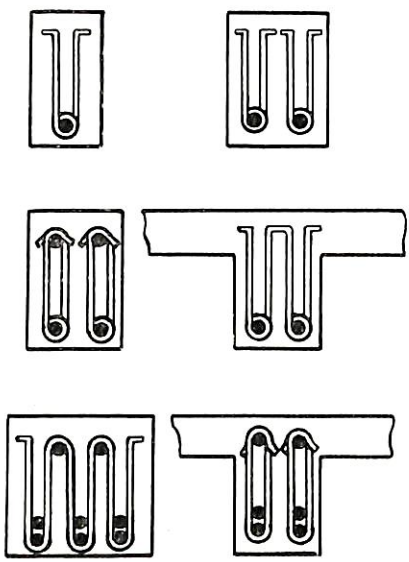


FIG 15.

Much controversy exists regarding the stress which it is permissible to develop in the stirrups. Some engineers contend that because the stresses are caused by shearing forces, the permissible stress in the stirrups should not exceed the shear resistance of the reinforcement, *i.e.* about 12,000 lbs. per sq. inch.

Others develop the full tensile resistance in the stirrups, urging that they are purely tensile members.

Again whilst some designers provide reinforcement to develop the whole of the diagonal tension, others only provide sufficient to develop the tension caused by the excess of shear above that which the concrete will resist, whilst others treat the materials as working in conjunction with each other and allow a stress in the reinforcement of m times that in the concrete.

The L.C.C. regulations demand that when the shearing stress, calculated on the concrete alone, is in excess of the permissible shearing stress, the **whole** shear shall be provided for by the tensile resistance of the shear reinforcement acting in conjunction with the compressive stresses in the rib.

The L.C.C. however do not definitely state the permissible stress to be developed in the shear reinforcement, but as they use the term " tensile resistance " it is presumed they will allow a stress of 16,000 lbs. per square inch. The author has often developed this stress in shear reinforcement, but would be reluctant to exceed it, although he might develop a greater stress in the main tensile reinforcement of the beam.

The diagonal stresses may be assumed to act in something like the manner they would in a steel lattice girder, as shewn in Fig. 16, the stirrups resisting the diagonal tension and the concrete the diagonal compression.

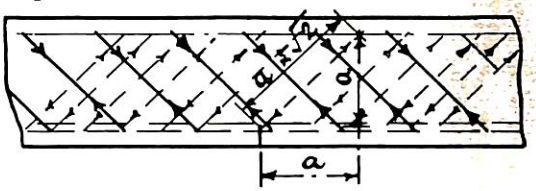


FIG 16. SHEWING STIRRUPS RESISTING DIAGONAL TENSION AND CONCRETE RESISTING DIAGONAL COMPRESSION

Therefore one-half of the shear forces at any point will be accommodated by the diagonal tension, and as the stirrups are inclined at 45 degrees to the horizontal the total diagonal tension on a length " a " equal to the lever arm of the beam will be $\frac{\sqrt{2}\,(S)}{2}$

This will be accommodated by the number " N " of stirrups in the length of " a."

Assuming that " p " = horizontal pitch of stirrups.
A_s = sectional area in square inches of all the legs in one stirrup.
t = permissible tensile stress per square inch in the stirrups.

The tensile resistance of the stirrups in a horizontal length " a" equal to the lever arm of the beam

$$= NA_s t = \frac{aA_s t}{p}$$ and equating this value with the diagonal shear,

$$\frac{aA_s t}{p} = \frac{\sqrt{2}(S)}{2} = \frac{S}{\sqrt{2}} \quad \therefore A_s = \frac{S}{\sqrt{2}} \times \frac{p}{at}$$

Example :—To design the stirrups for a Tee beam of 1 : 6 concrete in economic ratio, the load being 20 tons, evenly distributed. Effective depth = 15 inches, and breadth of rib " b_r " = 9 inches. First try whether the concrete will withstand the shear

$$S = \frac{W}{2} = 10 \text{ tons} = 22{,}400 \text{ lbs.}$$

Intensity of shear on concrete alone,

$$= s = \frac{S}{b_r a} = \frac{22{,}400}{9 \times 0{\cdot}88 \times 15} = \text{say } 186 \text{ lbs. per}$$

square inch, which is too great for the concrete alone. Therefore we shall have to introduce stirrups.

The required area of stirrup,

$$= A_s = \frac{S}{\sqrt{2}} \times \frac{p}{at}$$

Suppose we fix the pitch p of the stirrups at the point of maximum shear, *i.e.*, over the supports, at 6 inches, and that we use a value for t of 16,000 lbs. per sq. inch.

The required area of stirrup

$$= A_s = \frac{22{,}400}{1{\cdot}41} \times \frac{6}{0{\cdot}88 \times 15 \times 16{,}000} = 0{\cdot}45 \text{ square ins.}$$

Assuming there are six legs to each stirrup, the required area of the bar

$$= \frac{0{\cdot}45}{6} = 0{\cdot}075 \text{ square inches.}$$

A 5/16-in. diameter bar has an area of 0·076 square inches and would therefore be very suitable. Another method would be to assume a section of bar and then determine the required pitch.

As the shear diminishes from the points of support the pitch of the stirrups is increased, the increase being in direct geometrical progression, but it is not advisable to space the stirrups further apart than a length equal to the lever arm of the beam, otherwise the assumption of the angle of the diagonal forces would be wrong, also one must bear in mind that uneven loading is likely to take place in any beam.

The old-fashioned method of providing shear reinforcement was to place the stirrups vertically, and some firms even adhere to this method to-day, contending that the stirrups, especially if flat bars are used, can be more easily and more rigidly clipped to the longitudinal reinforcement. But this is a method which is practically obsolete, as the reinforcement is not used to theoretical advantage, because it is not placed across the natural line of failure. Further it is not so economical, because the assumption of the stirrups and concrete acting as a lattice girder with members inclined at 45 degrees to the horizontal gives place to a lattice girder of the ordinary " N " type. Therefore the whole of the shear on any point must be accommodated by the tensile resistance of the stirrups, and the tension for a length a would be equal to the shear. Therefore the required area of the stirrup when placed vertically

$$= A_s = \frac{S \times p}{at}$$

which would give greater results than when placed diagonally.

To limit the diagonal compression in the concrete, the L.C.C. regulations require that the vertical shear shall not exceed three times the shear resistance of the concrete measured over an area of the breadth × the effective depth of the beam.

$$\therefore \quad \frac{S}{3 \times b_r \times d} \text{ must not exceed the safe value of } s,$$

and the breadth of the rib must not be less than

$\frac{S}{3s \times d}$ which for our example would be $\frac{22{,}400}{3 \times 60 \times 15} = 8{\cdot}3$ ins.

and as our beam had a rib 9-in. wide, it would be quite in order.

When the longitudinal reinforcement is inclined across the neutral axis of the beam, as in the case of tensile reinforcement crossing over from the underside of the beam at a point of positive bending to the top of the beam at a point of negative bending, this reinforcement may be assumed to resist shear, and a reduction in the stirrups may be made. When this occurs, unless the angle of inclination of the longitudinal reinforcement is at 45 degrees to the horizontal, which is not often the case, allowance must be made for the increase in stress due to such deviation of the inclination.

DEFLECTION OF BEAMS.

To work any accurate formula for the deflection of a reinforced concrete beam is a practical impossibility. For beams where there is no change in the kind of bending moment, as in the case of a beam with freely supported ends, and where the section of the beam was unchanged throughout its length the formulae as applied to steel beams would be approximately applicable, but this is a case one seldom meets in practice. To be on the safe side, however, it is usual to limit the span of a beam measured centre to centre of supports to twenty times its effective depth. This is also the limit required by most of the authorities.

FOUNDATIONS.

In no other instance of this type of construction is the saving over ordinary methods so marked as in the case of foundation work.

The required depth or thickness of the foundations may be reduced by such a considerable extent by the use of a small amount of reinforcement, thereby reducing the cost of not only the concrete, but also the excavation work, which is of great importance, especially where the excavations are apt to become waterlogged.

The calculations for foundations are merely applications of the formulae for beams, and should offer no difficulties, with perhaps the exception of the foundations for pillars.

The foundation of a pillar resolves itself into a slab with a concentrated load, but the support is distributed over the whole of the underside of the slab. If we reinforce the slab in both directions, as shewn in Fig. 16, a quarter of the earth pressure may be assumed to act on each side of the pillar, as shewn by the triangles and the pressure on each triangle will work through its centre of gravity, *i.e.*, $\frac{L}{3}$ from the centre. Where L = length of slab in inches. Therefore the bending moment to be designed for in each direction is found from the formula :—

$$B = \frac{W_1}{4} \times \frac{2\,L}{3}$$

Where W_1 is the proportion of the load to be designed for in each direction as determined from the formulae for slabs. When the foundation is a square one, W_1 will be half the load W. Therefore in this case $B = \frac{W\,L}{12}$ (It will be noticed that the slight distribution of the load due to the width of the column shaft is entirely neglected).

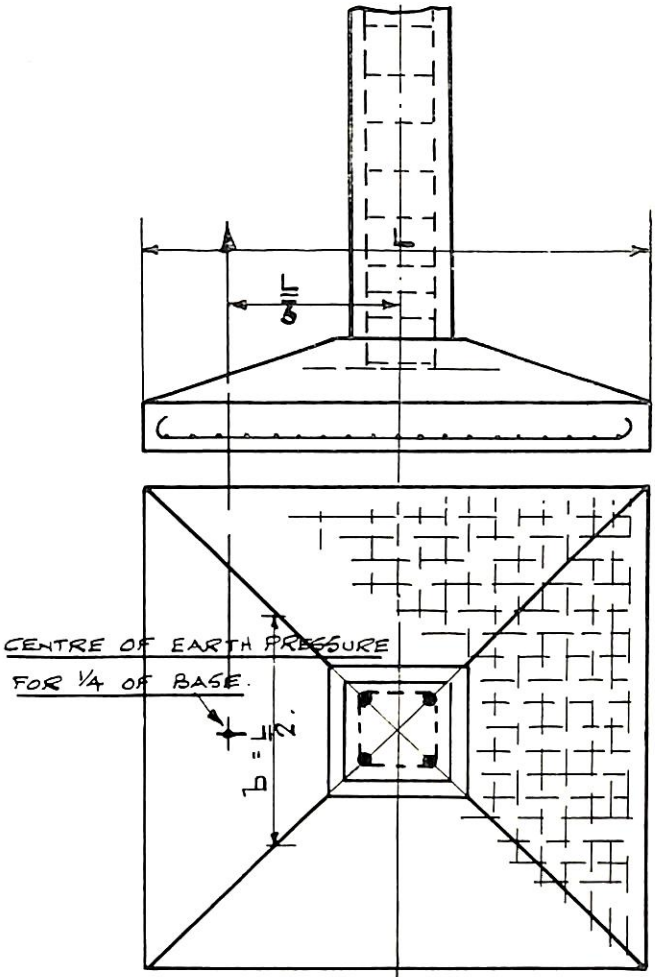


FIG 16.

The depth of the concrete, and consequently the effective depth of the slab is usually diminished towards the outer edge of the slab. To compensate for this reduction a mean depth of slab is used, or some engineers work from a mean breadth b, on the assumption that the four triangles act as inverted cantilevers. The result is the same in either case, and the latter case somewhat simplifies the calculations. It will be seen that in the case of a square base, the mean breadth b will be $\frac{L}{2}$ in both directions.

The shear diminishes from the centre of the base on account of the distribution of the earth pressure, consequently where shear reinforcement is required the pitch of the stirrups is increased accordingly.

Example :—To design the foundation of a pillar with a direct load of 54 tons, the mixture of the concrete being 1 : 6, and the safe earth pressure $1\frac{1}{2}$ tons per square foot, using tensile reinforcement in economic ratio

$$\text{Area of base required} = \frac{54}{1{\cdot}5} = 36 \text{ square feet} = 6 \text{ ft.} \times 6 \text{ ft.}$$

Bending moment to be designed for in each direction

$$= \frac{W\,L}{12} = \frac{54 \times 2240 \times 72}{12} = 725{,}760 \text{ inch pounds.}$$

$$b = \frac{L}{2} = 36 \text{ inches.} \qquad Q = 100. \qquad bd^2\,Q = 725{,}760.$$

$$\therefore\ d = \sqrt{\frac{725{,}760}{36 \times 100}} = 14{\cdot}2 \text{ inches.}$$

Economic area of tensile reinforcement =

$$0{\cdot}00675bd = 0{\cdot}00675 \times 36 \times 14{\cdot}2 = 3{\cdot}45 \text{ sq. inches.}$$

say 12 bars $\frac{5}{8}$-in. diameter which would be spaced over the whole width of the slab, *i.e.*, 12 bars in each direction.

The length of some of the bars could be curtailed as the bending moment diminishes from the centre, but except in the case of very large foundations it is not usual to do so, especially as the reinforcement is often in the form of meshing.

The shear would be accommodated exactly as for a beam or slab, half the load being taken in each direction.

$$\therefore\ S = \frac{W}{4} = \frac{54 \times 2240}{4} = 30{,}240 \text{ lbs.}$$

Intensity of shear stress s on concrete

$$= \frac{S}{ba} = \frac{30{,}240}{36 \times 0{\cdot}88 \times 14{\cdot}2} = 67 \text{ lbs. per sq. inch.}$$

Therefore shear stirrups will be required, which we will assume are inclined at 45 degrees. Assuming the stirrups to be $\frac{3}{8}$-in. diam., placed on six tension bars, each stirrup will have twelve legs. The area of one stirrup

$$= A_s = 12 \times 0{\cdot}11 = 1{\cdot}32 \text{ sq. ins.} \qquad \frac{aA_s t}{p} = \frac{S}{\sqrt{2}}$$

$$\therefore p = aA_s t \times \frac{\sqrt{2}}{S} = \frac{0{\cdot}88 \times 14{\cdot}2 \times 1{\cdot}32 \times 16{,}000 \times 1{\cdot}41}{30{,}240} =$$

say 12 inch pitch. Very often the stirrups in foundations are staggered. The compression on the concrete would be quite all right in our example because the value of b exceeds

$$\frac{S}{3 \times s \times d} = \frac{30{,}240}{3 \times 60 \times 14{\cdot}2} = 12{\cdot}3 \text{ inches.}$$

As a matter of fact except in the case of Tee beams it is seldom that the shear compression in the concrete need be taken into account.

The vertical reinforcement in the column shaft should be run into the foundation slab for a depth of at least six inches and then be turned outwards towards the corner of the slab for a distance of twenty times the diameter of the bar.

Very often compressive reinforcement is introduced to further reduce the depth of the foundation slab, and even when this is not taken into account in the calculations most designers include two or three bars in the compression area in each direction to act more as distributing bars than compressive reinforcement.

An alternative method of designing a column foundation is shewn in plan view in Fig. 16A. This is a method which is largely adopted in America, and one which the author considers more sound in principle than the foregoing method, although not so widely adopted in this country. The reinforcement is arranged as shewn in the diagram, in such a manner as to form eight inverted cantilever brackets with an assumed breadth b equal to a quarter of the length of the slab in each case. To simplify calculations each cantilever is assumed to resist one-eighth of the load, and the bending moment on each cantilever is assumed to be $\frac{W\,L}{24}$

This method is also very applicable to columns with round bases instead of the square type.

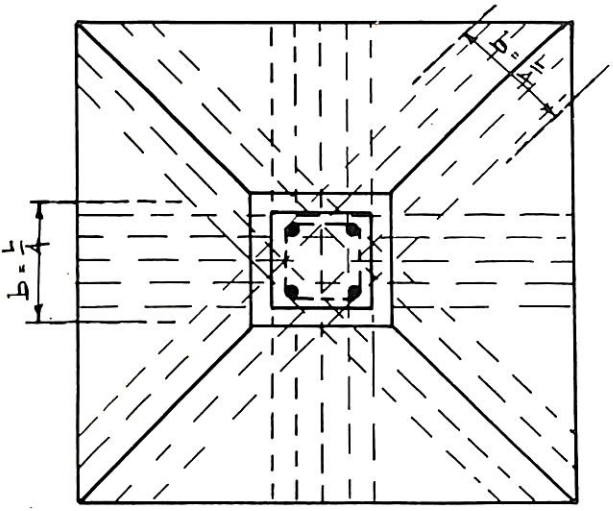


FIG 16A.

The cover for foundation reinforcement should not be less than two inches, nor less than twice the diameter of the bar, although very often this cover is made much larger on the underside of the slab.

When overturning moments are likely to occur, or when the load on a pillar is not assumed to be evenly distributed over its base, allowance must be made for the eccentricity of the centre of pressure.

WALLS.

More often than not, reinforced concrete walls are merely panels between the main columns and floor beams of the building, and are treated as such, being designed to resist the wind pressure as an ordinary slab. To comply with the L.C.C. regulations, however, the total thickness of an external reinforced concrete wall must not be less than 4 inches (the minimum allowable thickness for external brick walls is given as $8\frac{1}{2}$ inches).

Where the walls support floor, roof, or any other loads, due regard must be taken of same in the calculations.

Although the author feels that much that he would have liked to include in this article has been omitted, he is of the opinion that, by the application of the foregoing formulae, along with ordinary static principles of engineering, one should be able to go a long way in the design of most types of reinforced concrete construction.

SUMMARY OF FORMULAE.

Working Stresses in Material (L. C. C. Limits).

Safe compressive resistance of plain concrete $c = 900 - 50\,V$ (See table on page 5).

,, shear ,, ,, $= \frac{c}{10}$

,, tensile ,, mild steel, = 16,000 lbs. per sq. inch

Modulus of elasticity of concrete $= 3{,}333\frac{1}{3}\,c$ lbs. per sq. inch.

,, ,, mild steel = 30,000,000 ,, ,,

Modular ratio $m = \frac{9000}{c}$ = 15 for 1 : 6, 13·85 for 1 : 5, 12·86 for 1 : 4 and 12 for 1 : 3 mixtures of concrete respectively.

Columns with Simple Reinforcement.

$$P = c\,[A + (m-1)\,A_v] = c\,A\,[1 + (m-1)\,r_v]$$

$$\text{or Reqd. } A = \frac{P}{c\,[1 + (m-1)\,r_v]}$$

Columns with Hooped Core.

$i = c\,(1 + f\,s\,V_1)$ $\quad s = 48 - 80\,\frac{P_b}{d}$ See table on page 12.

$$P = i\,[A + (m-1)\,A_v] = i\,A\,[1 + (m-1)\,r_v]$$

$$\text{or Reqd. } A = \frac{P}{i\,[1 + (m-1)\,r_v]}$$

$$V_1 = \frac{4\,A_b}{d\,P_b} \quad \text{or} \quad A_b = \frac{V_1\,d\,P_b}{4}$$

Columns subject to Bending Stresses.

$$f = f_d + f_b = \frac{P}{A\,[1 + (m-1)r_v]} \pm \frac{B}{Z}$$ for values of I and Z, see pages 14 and 16.

Long Columns.

$$g = \sqrt{\frac{I}{A + (m-1)\,A_v}} = \sqrt{\frac{I}{A\,[1 + (m-1)\,r_v]}}$$

See tables on page 18.

Bending Moments. See tables on pages 21 and 22.

Single Reinforced Rectangular Beams and Tee Beams (Cases 1 and 2) where Neutral Axis is not below bottom of Slab.

$$n_1 = \sqrt{2\,r\,m + r^2 m^2} - r\,m.$$

Value of r to give economic ratio $r = \dfrac{c\,^2 m}{2\,t\,(t + c\,m)}$

$R = b\,d^2\,Q.$ $\quad Q = rt\left[1 - \dfrac{n_1}{3}\right]$ when working from tensile values and

$Q = \dfrac{c\,n_1}{2}\left[1 - \dfrac{n_1}{3}\right]$ when working from compressive values.

For values of n_1, r and Q for beams of varying mixtures in economic ratio, see table on page 30.

Tee Beams (Case 3) when Neutral Axis falls below bottom of Slab.

$$n_1 = \frac{s_1^2 + 2\,m\,r}{2\,(s_1 + m\,r)}.$$

Value of r to give economic ratio $r = \dfrac{2\,m\,c\,s_1 - m\,c\,s_1^2 - t\,s_1^2}{2\,m\,t}$

$R = bd^2\,Q.$ $\quad$ *Q when working from tensile values

$$= t\left[\frac{s_1^3 + 4\,m\,r\,s_1^2 - 12\,m\,r\,s_1 + 12\,m\,r}{6\,m\,(2 - s_1)}\right]$$

Q when working from compressive values =

$$cs_1\left[\frac{s_1^3 + 4\,m\,r\,s_1^2 + 12\,m\,r\,s_1 + 12\,m\,r}{6\,(s_1^2 + 2\,m\,r)}\right]$$

* Usually the lower value of Q.

Slabs.

$$W_1 = \frac{W}{1 + \left(\frac{B}{L}\right)^4} \text{ and } W_2 = \frac{W}{1 + \left(\frac{L}{B}\right)^4}$$

Shear in Beams.

$$s = \frac{S}{b_r\, a}$$

$$\text{Reqd. } A_s = \frac{S}{\sqrt{2}} \times \frac{p}{a\, t} \text{ or}$$

$$\text{Reqd. } p = A_s\, a\, t \times \frac{\sqrt{2}}{S} \text{ when stirrups inclined at 45 deg.}$$

$$\text{Reqd. } A_s = \frac{S \times p}{a\, t} \qquad \text{or Reqd. } p = \frac{A_s\, a\, t}{S}$$

when stirrups vertical.

SUMMARY OF NOTATION.

A = Effective area (for columns) and total area of tensile reinforcement for beams.

A_b = Area of one binder or lateral reinforcing bar.

A_c = Total area of compressive reinforcement in beam.

A_s = Area of all legs in one stirrup in beam.

A_v = Total area of vertical reinforcement in column.

a = Arm of moment of resistance of beam.

B = Bending moment of external forces in inch pounds.

B = Breadth of slab (when determining portion of load to be carried in each direction).

b = Breadth of beam.

b_r = Breadth of rib of Tee beam.

c = Permissible safe compressive stress in plain concrete, for columns.

c = Actual compressive stress in plain concrete at edge of beam.

d = Effective diameter or width of column (measured to outside of vertical reinforcement).

d = Effective depth of beam, *i.e.* distance from compressive edge of concrete to centre of area of tensile reinforcement.

d_c = Distance of centre of compressive reinforcement from the compressive edge of beam.

d_s = Depth of slab in inches.

f = Form factor which varies for different types of lateral reinforcement for columns.

f = Combined direct and bending stress at edge of column, $\therefore f = f_d + f_b$

f_b = Bending stress in column (worked from an equivalent section modulus of concrete).

f_d = Direct stress in column (worked from an equivalent sectional area of concrete).

g = Radius of gyration.

h_s = Distance of vertical reinforcement from axis of column.

I = Inertia of section.

i = Increased safe resistance of hooped core of concrete.

l = Length of axis of column or length of effective span of beam or slab.

L = Length of slab (when determining portion of load to be carried in each direction).

m = Modular ratio or ratio of modulus of elasticity of steel to that of concrete.

N = Number of stirrups in length of beam equal to lever arm of moment of resistance.

n = Distance of neutral axis from compressive edge of beam.

n_1 = Ratio of $n : d$ in beams $\therefore n_1 d = n$.

P = Permissible load on short columns or equivalent permissible load on long columns.

p = Horizontal pitch of stirrups in beams.

P_b = Pitch of lateral reinforcement or binders in columns.

Q = Qualifier in the equation $R = Q\,b\,d^2$ for moment of resistance of beams.

R = Moment of resistance of beam in inch pounds.

r = Ratio of area of tensile reinforcement to effective area of beam $\therefore r\,b\,d = A$.

r_1 = Ratio of area of compressive reinforcement to effective area of beam $\therefore r_1\,b\,d = A_c$

r_v = Ratio of area of vertical reinforcement to effective area of column $\therefore r_v A = A_v$

S = Total shear on beam.

s = Intensity of shear stress in concrete of beams, slabs, etc.

s = Spacing factor for lateral reinforcement of columns.

s_1 = Ratio of depth of slab to effective depth of Tee beam

$$\therefore s_1 = \frac{d_s}{d}$$

t = Stress in tensile reinforcement.

V = Volume of sand and ballast per unit volume of cement in concrete.

V_1 = Ratio of volume of lateral reinforcement to volume of hooped core of column.

v = Virtual length of column.

W = Total load.

W_1 = Portion of load to be carried in direction of width of slab.

W_2 = Portion of load to be carried in direction of length of slab.

Z = Modulus of section.

Note.—All Loads in Pounds, all Dimensions in Inches, all Areas in Square Inches, and all Stresses in Pounds per Square Inch.